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From Transformity to Ordinality, or better: From Generative Transformity to Ordinal Generativity

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ABSTRACT

The paper starts by recalling the three Generative Rules of Emergy Algebra (Co-production, Inter-action, Feed-back) in order to show a possible "hiatus" between the concepts understood (by Prof. Odum) in terms of "verbal enunciations" and their corresponding "mathematical translations", that is the formal symbology habitually adopted to express them.

On the basis of such a hypothesis, and in the perspective of generalizing the previous Rules under dynamic conditions (where such a supposed discrepancy would have surely become much more marked), a different concept of derivative was introduced (the so-called "incipient" derivative). This allowed us to remove, from the very beginning, such an initial "in-harmony" in those definitions, in which radically new concepts are directly expressed by means of old mathematical symbolisms. The concept of "incipient" derivative was the one which led us to the first "transition" indicated by the title: Transformity, in fact, can more effectively be replaced by the concept of Ordinality.

However, further research on the fundamental role played by Generative Transformity (with respect to Dissipative Transformity) suggested an even more generalized approach to the analysis of Self-Organizing Systems. This gave origin to the second successive "transition" (also synthesized in the title): Generative Transformity, in fact, can profitably be replaced by Ordinal Generativity.

Some Ostensive Examples, taken from Classical Mechanics, Relativistic Mechanics and Quantum Mechanics, will then clearly show how the latter approach is able to realize Odum's "dream" concerning "the Unification of Science", when based on a System perspective, Energy hierarchy and the fundamental concept of Emergy.[1]

INTRODUCTION

When Prof. H. T. Odum introduced the concept of Emergy (Odum 1988), such a proposal divided the scientific community in two well-distinct basic classes which, for the sake of brevity, could be termed as the Class of "NO" and the Class of "YES", respectively. The former is made up of those Scientists who *do not believe* Emergy represents a true novelty in Thermodynamics. The latter, on the contrary, is made up of those Scientists *who do believe* that Emergy represents a *real novelty* in Thermodynamics.

Here we will not explicitly deal with the former Class. Nonetheless, the analysis devoted to the latter will be sufficient, in itself, to answer the most frequent criticisms addressed to the approach proposed by Prof. Odum.

[1] Odum H. T., 1995. *Energy Systems and The Unification of Science*. University Press of Colorado, pp. 365-372.

The Class of "YES", in turn, can be articulated in three distinct sub-Classes, according to the *formal language* adopted to translate, into mathematical terms, the verbal enunciations of the basic Rules of Emergy Algebra (Brown 1993; Brown and Herendeen 1996). In fact:

i) the First Emergy sub-Class interprets Emergy Rules as a *pure non-conservative Algebra*;

ii) the Second sub-Class interprets Emergy Rules as an *Ordinal non-conservative Algebra* (that is, an Algebra capable of "vehicling" an intrinsic *Ordinal meaning*);

iii) The Third sub-Class interprets Emergy Rules as being a verbal expression of *Ordinal Differential Equations*.

We will now examine, in rapid sequence: i) the basic differences (between the three sub-Classes) at a *linguistic-mathematical* level; ii) the *subjacent conceptual* differences (in terms of the corresponding presuppositions assumed); iii) the main different *consequences* in practice (that is, in terms of results).

To this purpose, it is worth recalling a deep distinction which subsists between those same Rules of Emergy Algebra (Giannantoni 2006a).

Rules of Genesis and Transfer of Ordinality

The Rules of Emergy Algebra, in fact, can be subdivided in *two groups* and re-proposed in a different *sequence*, by always keeping the same formulation given by Prof. Brown (Brown 1993; Brown & Herendeen 1996)):

1st group: made up of Co-production, Inter-action and Feed-back. These can be seen as *Rules of Genesis of Ordinality* (in steady state conditions):

i) Co-production**:** "*By-products from a Process have the total Emergy assigned to each pathway*"

ii) Inter-action: "*Output Emergy of an interaction Process is proportional to the product of the Emergy inputs*" (Odum, 1994a)

iii) Feed-back**:** *"Emergy in feedbacks should not be double counted* ".

2nd group: made up of the First Rule, Split Rule, and the Rule of Recombination of By-products. These can be seen as *Rules of Transfer of Ordinality* (in steady state conditions), either from one part of the System to another or at a local level. In particular: from input to output (the former rule); from the main flow to the subdivided flows and vice versa (the latter rules):

iv) First Rule: "*All Source Emergy to a Process is assigned to the Process's output* "

v) Split: *"When a pathway splits, the Emergy is assigned to each "leg" of the split based on their percent of the total Exergy flow on the pathway"* (ib.; Giannantoni, 2002).

vi) By-products "*By-products, when reunited, cannot be summed* " (Brown, 1993)

However, in order to show the differences between the three afore-mentioned Emergy sub-Classes it is not necessary to analyze all the different Rules. One of them is already sufficient, because the subjacent Logic is always the same. We can thus consider, for example, the Co-production Rule (a wider analysis of all the Rules of Emergy Algebra is given in (Giannantoni 2006a, 2007a,b)).

BASIC DIFFERENCES BETWEEN THE THREE EMERGY SUB-CLASSES

The First Emergy Sub-Class is characterized by an "assent" to the verbal definition of Emergy Rules in "*strictly algebraic"* terms. In fact, the basic assumption is that the pertinent verbal definition (*"By-products from a Process have the total Emergy assigned to each pathway"*) can simply be translated in formal terms as follows

$$Em(y_1) = Em(u) \quad (1) \qquad\qquad Em(y_2) = Em(u) \quad (2)$$

where the symbol of equality (=) is understood in its proper *algebraic* sense (together with all its *logical*, *causal* and *functional* implications (see also Appendix)). Thus it is also coherent to write

$$Em(y) = 2 \cdot Em(u) \qquad (3) \qquad\qquad Em = Tr \cdot Ex \qquad (4)$$

where Tr is the Transformity of the Co-production Process.
Let us now consider the corresponding consequences of such an approach, both under steady state and dynamic conditions:

Under *steady state* conditions: i) Eqs. (3) and (4) lead to a generalization of the previous concept of Embodied Energy; ii) this presents substantial advantages in Economic Analyses, in particular when assigning Economic Values to Natural Resources (since Economics, as is well known, does not recognize any value to them unless they enter the "market"); iii) such an advantage is especially due to a sort of "amplification effect" (associated to the Rules of Emergy Algebra; see, for instance, Eq. (3)) which favors a more appropriate allocation of resources; iv) apart from the additional advantage of a direct adoption of well-known mathematical tools (e. g. Linear Operators);

Under *dynamic* conditions: i) there is the advantage of performing the pertinent analysis in terms of Traditional Differential Calculus***;*** ii) such an option, in fact, is *perfectly conform* to the basic assumption of a "strictly algebraic" assent***;*** iii) with the additional advantage of a possible adoption of numerical methods already available*;* iv) even though solutions in this way obtained are generally reliable only under linear (or linearized) conditions.

In essence, the First Emergy sub-Class follows a theoretical approach which is: a) extremely *coherent*; b) even if, for some aspects, it could appear as being a *conventional* accountability technique (and this is sometimes source of some criticism).

The Second Emergy Sub-Class is characterized by an "assent" to the verbal definition of the Emergy Rules in terms of an *implicitly Ordinal Algebra*. In fact, the afore-mentioned verbal enunciation (*"By-products from a Process have the total Emergy assigned to each pathway"*) is still "translated" according to Eqs. (1) and (2). These, however, are interpreted differently. In fact, Scientists pertaining to this sub-Class bring out the fact that the verbal definition does not literally state "…*is equal to*", but, on the contrary, it simply says "…*is assigned to*". This is why the sign of equality (=) is not understood in its algebraic sense, but only as a *symbol* of an *assignation* (or *attribution*) of quantities. Eqs. (1) and (2) should be thus formally written in a different way. For example, as follows

$$Em(y_1) \stackrel{*}{=} Em(u) \qquad (5) \qquad\qquad Em(y_2) \stackrel{*}{=} Em(u) \qquad (6)$$

where the symbol $\stackrel{*}{=}$ would remark such a difference.
The same considerations can evidently be referred to Eq. (4), which now becomes

$$Em \stackrel{*}{=} Tr \cdot Ex \qquad (7),$$

where Transformity Tr is now correspondently interpreted as a "cipher", understood in its proper gnoseological sense.[2] Such an explicit notation, however, is not adopted systematically. The Analyst, although "thinking" in terms of Eqs. (5), (6), (7), continues to "write" them in the traditional forms (1), (2), (5). In this way he "thinks" he has delegated to such formal (algebraic) representation the *mere* "task" of guiding his analysis, in hoping to "recuperate" the Ordinal sense of the same at the end of such a mathematical process. Such an assumption, however, is not rigorously correct, because it is characterized by an intrinsic "irreversibility". This can simply be shown by the related consequences, both under steady state and dynamic conditions:

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[2] "*cipher* " (in gnoseology) is any symbol, of a *given* nature, adopted to represent another entity, of a completely *different* nature.

Under *steady state* conditions: there is a sort of "leveling" effect, which is very similar to the "leveling" of Temperature in heat transfer, in the presence of several thermal sources, with an "analogous" lost of Ordinal Information (an "Entropy" effect). Such an effect is mainly due to: i) the absence of an explicit notation of the *ordinal sense* of the mentioned equations; ii) in particular, for that component of Transformity which is specifically associated to a Generative Rule; iii) the effect manifests itself in a progressive "reduction" of both *inner* and *output* Transformity values; iv) and the effect is progressively more marked according to the *dimensions* of the Process/Plant analyzed.

This can easily be shown by remembering that Transformity (*Tr*) can always be articulated in two factors (Giannantoni 2002, p. 64 , 2006a)

$$Tr = Tr_{\phi} \cdot Tr_{ex} \qquad (8)$$

where Tr_{ex} (dissipative Transformity) accounts for the losses of Exergy used up during the generation process of a given product or service, whereas Tr_{ϕ} (generative Transformity) accounts for the ever-increasing content of *Ordinal Information* due to those generative Processes. Consequently: i) while the *global* Transformity (*Tr*) (see Eq. (8)) may reach a very high value; ii) the corresponding generative Transformity Tr_{ϕ} will progressively tend to assume lower values (always greater than 1, but usually very close to 1 in the case of very big Plants).

Under such conditions, in fact, the value of Emergy output of a Process is mainly due to the contribution of the Total Exergy spent ($Tr_{ex} \cdot Ex$). In this sense Emergy output can be considered substantially "proportional" to Total Exergy spent, according to a *dimensional* coefficient K_D (sej/J), which has an almost stable value (ranging from $1.1 \div 1.2$) for all the Plants characterized by very wide geometrical dimensions (let us think, for example, of a nuclear power plant of 1000 MW). This is because the dimension of the Plant can be assumed as an Indicator of the increasing number of Emergy transfer processes between components, in order to yield the generation of one (or more) products. The *loss of Information*, in fact, can be basically localized at the level of Transfer Rules, exactly because such a "transfer" is formally represented through an *un-explicit* Ordinal notation for the involved Emergies and Transformities. Vice versa, the adoption of an explicit Ordinal notation for the latter would overcome such a limitation, by transforming Emergy Analysis into the *First Ordinal Theory of Complex Systems* (Giannantoni 2006b).

Under *dynamic* conditions: the fact that Emergy Analysis is usually performed under very slow transients generally prevents another additional effect. In fact: i) the usual adoption of TDC introduces a potential "drift" effect associated to the same concept of the traditional "a posteriori" derivative (Giannantoni 2004b; see also Tab. 3); ii) without mentioning the fact that the same adoption of TDC would be in open contrast with the basic assumption (concerning the Ordinal meaning of the adopted equations). In fact, the traditional derivative *intrinsically filters* any form of Ordinality, because it is an expression of a mere *cardinal* concept (see also Tab. 1 and Tab. 3, later on; see also Appendix).

In essence, the Second Emergy sub-Class adopts a theoretical approach which: i) works much better in *steady-state* (or pseudo-dynamic) conditions; ii) it is however prevailingly reserved to *very experienced* Analysts. These, in fact, on the basis of their experience, almost always succeed in "compensating" the above-mentioned "leveling" effect (albeit with the persistence of some related "risks").

The Third Emergy Sub-Class is characterized by an "assent" to the verbal definition of the Emergy Rules in *Ordinal Differential* terms (more precisely, in terms of *Incipient Differential Calculus* (IDC)). Such an approach, in fact, is exactly the same as that which originated from the formalization process of Emergy Algebra under dynamic conditions and, subsequently, from the mathematical formulation of the Maximum Em-Power Principle (Giannantoni 2001a,b,c, 2002, 2004a,b). In this perspective, in fact, the Rules of Emergy Algebra were first generalized to dynamic conditions (Giannantoni 2002) through the introduction of a new concept of derivative (the "incipient" derivative) (ib., p. 175). Afterwards, the availability of a new mathematical language (represented by IDC) progressively revealed that Odum's Rules are, in reality, much *more profound* than we had ever thought. Their verbal definition, in fact, can be considered as being already given under *dynamic* conditions, because *Emergy Rules* specifically refer to Processes which are *intrinsically Generative*.

As an example, the same afore-mentioned Co-production definition (*"By-products from a Process have the total Emergy assigned to each pathway"*) can now be translated into Eq. (9), that is in term of "binary functions" (see Giannantoni 2002, p. 173; 2004a,b)

$$\tilde{E}m(u) \overset{[\rightarrow}{=} \begin{pmatrix} \tilde{E}m(y_1) \\ \tilde{E}m(y_2) \end{pmatrix} \qquad (9)$$

Eq. (9), in fact, is not a traditional cardinal functional relationship, but an *Ordinal Relation* (see Appendix), in which the symbol " $\overset{[\rightarrow}{=}$ " does not represent a simple "equality". It indicates, on the contrary, a "jump" in Emergy output Ordinality, which is, however, always "adherent" (" $[\rightarrow$ ") to its premises. Correspondently, the *usual* conditions (1) and (2) (here reproposed for the sake of clarity)

$$Em(y_1) = Em(u) \qquad (1)$$

$$Em(y_2) = Em(u) \qquad (2)$$

now appear as being a *simple adherent reflex* of the *interior properties* of the incipient derivative of Order ½. This also clarifies the sense of the *assignation* concept expressed by Eqs. (5) and (6) (here reproduced for a more direct comparison):

$$Em(y_1) \overset{*}{=} Em(u) \qquad (5)$$

$$Em(y_2) \overset{*}{=} Em(u) \qquad (6).$$

On the basis of such premises we can now recall the main reasons for introducing a new concept of derivative.

Adoption of a new concept of derivative

The introduction of a new concept of derivative was essentially suggested by the same Rules of Emergy Algebra, exactly because the latter try to express the real novelty represented by the three fundamental *Generative Processes* (Giannantoni 2004b). These, in fact, through the original concept of Emergy, introduce a profound novelty in Thermodynamics. The fact that: *there are processes, in nature, which cannot be considered as being pure* "*mechanisms*". This is equivalent to say that they are not describable in mere functional terms, because their outputs show an unexpected "excess". The latter can be termed as Quality (with a *capital* Q) exactly because it is no longer understood as a simple "property" or a "characteristic" of a given phenomenon, but it is recognized as being an *emerging* "property" (from the considered process) *never ever reducible* to its phenomenological premises or to our traditional mental categories (Giannantoni 2002, p. 96; 2004d)[3].

[3] This is also the reason for the corresponding Capitalization of all the fundamental terms (substantives, adjectives, verbs) when re-interpreted in the light of such a new concept of Quality.

Table 1 - Synoptic Comparison between the Basic Presuppositions pertaining to TDC and IDC

Traditional Differential Calculus	*Incipient* Differential Calculus
1) efficient causality	1') Generative Causality
2) necessary logic	2') Adherent Logic
3) functional relationships	3') Ordinal Relations

The attempt to represent such an output "excess" was the one that then led us to the introduction of a new concept of derivative.

The traditional derivative, in fact, is not able to represent any form of "excess", because it is the direct "formal translation" of the three basic presuppositions usually adopted when describing the surrounding world: *efficient* causality, *necessary* logic, *functional* relationships (see Tab. 1).
Generative Processes, on the contrary, and the pertinent Emergy Rules (which represent them), suggest a different form of "causality", in a strict adherence to the fact that process outputs are characterized by a higher level of Quality with respect to their inputs. This new form of Causality can be termed as *Generative* Causality (*Source* Causality, *Spring* Causality and so on) (Giannantoni 2002, p. 103; see also Appendix). In all cases the associated concept (previously pointed out) is clear.

The same can be said as far as Logic is concerned. "Necessary" Logic, in fact, is not able to foresee any "excess" in the conclusions with respect to the corresponding premises. This is why "necessary" Logic is better replaced by "*Adherent* Logic". That is a Logic where conclusions may be richer than their corresponding premises. (ib.; see also Appendix). Analogously, *functional* relationships will be adherently replaced by *Ordinal* Relationships (ib.; see also Appendix).

Let us now recall the definition of the Incipient Derivative in order to highlight its fundamental properties.

The "incipient" derivative and the associated IDC

The incipient derivative ($\tilde{d}/\tilde{d}\,t$) is defined as follows (Giannantoni 2002, p. 175):

$$(\frac{\tilde{d}}{\tilde{d}\,t})^q f(t) = \underset{\tilde{\Delta}t:0\to 0^+}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t}\right)^q \circ f(t) \qquad \text{for any} \quad q \in Q \qquad (11)$$

where:

i) the *"operator"* $\tilde{\delta}$ *generates* the "translation" $\tilde{\delta}\, f(t) = f(t+\tilde{\Delta}t)$; ii) the sequence of symbols is interpreted *from left to right*; iii) the sequence is also interpreted as a *generative inter-action* between the *three* considered concepts (see the symbol "o"), iv) the definition is valid for *any integer or fractional Order* $q = m/n$. In fact, by remembering that any function $f(t)$ can always be structured in the exponential form $f(t) = e^{\ln f(t)} = e^{\alpha(t)}$, definition (11) leads to

$$(\frac{\tilde{d}}{\tilde{d}\,t})^{m/n} e^{\tilde{\alpha}(t)} = [\overset{\circ}{\alpha}(t)]^{(m/n)} \circ e^{\tilde{\alpha}(t)} \qquad (11'),$$

where $\overset{\circ}{\alpha}(t)$ is the first order incipient derivative of the exponent $\alpha(t) = \ln f(t)$ (see Appendix).
Such a new form of derivative, exactly because of its special properties, seems to be particularly indicated to express Emergy Concepts. Consequently, the associated *Incipient* Differential Calculus

(IDC) appears to be much more appropriate than the Traditional Differential Calculus (TDC), when dealing with Emergy Systems Dynamics.

As previously said, the introduction of the incipient derivative was initially conceived as a simple support, of a *linguistic nature*, to Emergy Analysis, so as to generalize the Rules of Emergy Algebra from "steady-state" to "dynamic" conditions. This, in fact, would have certainly represented a fundamental step in order to give the most general Mathematical Formulation to the Maximum Em-Power Principle (Giannantoni 2001b). However, after having achieved such a general formulation (Giannantoni 2002), the incipient derivative (and the associated IDC) enabled us to recognize that Odum's Rules are much *more profound* than we had ever thought. In actual fact, only on the basis of such a formal language did it become possible to show that *such Rules, although formulated in phenomenological terms, directly refer to the most intimate Genarativity of the Processes.*

This was possible exactly because a *Formal Language* is not only an expression of thought, but also (and especially) a *support* (and even a "*guide*") to our thought.

FROM TRANSFORMITY TO ORDINALITY

In the first phase of generalization of the formal description of Generative Processes, the incipient derivative allowed us to show that Generative Transformity (Tr_ϕ) can profitably be replaced by Ordinality, that is the Ordinal Power of the incipient derivative (Giannantoni 2002, 2006a, 2007a,b). This also led, as an immediate consequence, to the proposal to adopting an explicit Ordinal notation for Generative Transformity $(\tilde{T}\, r_\phi)$, precisely as *emerging* from the IDC, so as to have

$$Tr = [(\tilde{T}\, r_\phi), Tr_{ex}] \qquad (12).$$

In this way, in fact, Emergy Analysis would become the *First Ordinal Theory of Complex Systems* (ib.)

However, further research on the fundamental role played by Generative Transformity (with respect to dissipative Transformity Tr_{ex}) suggested an even more generalized approach to the analysis of Self-Organizing Systems.

FROM GENERATIVE TRANSFORMITY TO ORDINAL GENERATIVITY

In fact, by taking into account that under *Generative Dynamic* Conditions [4]:

Co-production yields "binary" functions: $$\left(\frac{\tilde{d}}{\tilde{d}\,t}\right)^{\frac{1}{2}} e^{\alpha(t)} = \begin{pmatrix} +\sqrt{\overset{\circ}{\alpha}(t)} \\ -\sqrt{\overset{\circ}{\alpha}(t)} \end{pmatrix} \cdot e^{\alpha(t)} \qquad (13)$$

Inter-action yields "duet" functions: $$\left(\frac{\tilde{d}}{\tilde{d}\,t}\right)^{2} e^{\alpha(t)} = \left[(\overset{\circ}{\alpha}(t))^2, (\overset{\circ}{\alpha}(t))^2 \right] \cdot e^{\alpha(t)} \qquad (14)$$

Feed-back gives "duet-binary" functions: $$\left(\frac{\tilde{d}}{\tilde{d}\,t}\right)^{\frac{2}{2}} e^{\alpha(t)} = \left[\begin{pmatrix} +\overset{\circ}{\alpha}(t) \\ -\overset{\circ}{\alpha}(t) \end{pmatrix}, \begin{pmatrix} +\overset{\circ}{\alpha}(t) \\ -\overset{\circ}{\alpha}(t) \end{pmatrix} \right] \cdot e^{\alpha(t)} \qquad (15),$$

[4] Eqs. (13), (14), (15) are written with reference to exponential function because, as previously recalled, any function $f(t)$ can always be structured in such a form.

we can easily recognize that, while the right hand sides of Eqs. (13), (14), (15) represent the Ordinal structure of Co-production, Inter-action and Feed-back Processes, respectively, the corresponding left hand sides have an identical structure, always in the form $(\tilde{d}/\tilde{d}\,t)^q$, where q is a rational number which assumes the values of 1/2, 2 and 2/2, respectively. Now, by considering that the incipient derivative $\tilde{d}/\tilde{d}\,t$ is the most appropriate mathematical concept to express the Generative Activity of a Self-organizing System, we can easily recognize that all Generative Processes are characterized by the same Generativity, which, however, can assume different forms, according to the Ordinality *q*. We can thus assert that the concept of Generative Transformity is a direct faithful Reflex of an Ordinal Generativity. That is, a Generativity of Ordinal nature, because characterized by a specific Ordinality since the very beginning of the Process (see also Appendix).

SOME CONSEQUENCES OF THE ORDINAL DIFFERENTIAL APPROACH

The most important consequences in some Disciplines (in particular, Classical Mechanics, Relativistic Mechanics and Quantum Mechanics) have already been synthetically anticipated in (Giannantoni 2006a) and, afterwards, much more widely analyzed in (Giannantoni 2007b)[5]. In the latter work three specific examples for each Discipline were selected in order to show the wide relevance of such a new Differential Approach. This, on the other hand, is simply founded on the Rules of Emergy Algebra, when the latter are interpreted as an expression, by themselves, of the "Generative" Dynamics pertaining to Self-Organizing Systems. For the sake of brevity, we will here recall the results pertaining to three examples only (one for each mentioned Discipline).

Classical Mechanics: the Three-body Problem

The Three-body Problem was proved to be *intrinsically unsolvable* in Classical Mechanics (Poincaré 1899). In fact it is described by an 18th-order system of ordinary differential equations which, however, admits only 2 first order *closed form* integrals (energy and areas) (ib. , vol. 1, p. 253)[6] Vice versa, when faced in terms of incipient derivatives, the problem becomes perfectly solvable, in the sense that: there exists *at least* one solution in *a closed form*, as explicitly desired by Poincaré (ib.). What's more, such a solution can be easily obtained (always in *a closed form*) and at three different levels of Ordinality (Giannantoni 2007b, pp. 49-60). The main reason for the afore-mentioned solution depends on some basic differences between *Incipient* derivative and *Traditional* Derivative, which can here simply be recalled and synthetically illustrated in Tab. 3 (see also Giannantoni 2007b, pp. 33, 43). Such properties, although referred to the exponential function $e^{\alpha(t)}$, are valid for any considered function $f(t)$, always for the same reason previously mentioned.

A direct comparison between the different consequences corresponding to the two well-distinct approaches is shown in Tab. 2, whereas the fundamental bases of the results up to now achieved are synthetically recalled in footnote [7].

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[5] Some important consequences in Electromagnetism, Chemistry, Biology and Cosmology will be shown in a new book which will be published in next spring.

[6] To quote the same Poincaré : "…*le problème de trois Corps n'admet pas d'autre intégrale uniforme que celle des force vives et des aires*. " (ib.), where the concept of "integral" is not simply understood according to the traditional sense of "solution", but as a "function of solutions" (ib., p. 8) structured in the form $F_i[x_1(t), x_2(t), \ldots x_n(t)] = cost$, where $x_1(t),\ x_2(t), \ldots x_n(t)$ represent the generic unknown variables of the considered problem.

[7] The fact that the "Three-body problem", even in its *most general form*, admits at least *one solution in a closed form* when reformulated in IDC, is substantially due to the *intrinsic* and *specific* properties of the *incipient derivatives* (see Appendix). In fact such a solution can be obtained on the basis of the following : i) the *Fundamental Theorem of the Solving Kernel* (Giannantoni, 1995), which gives the

Table 2 - New Results in Classical Mechanics

Traditional Differential Calculus	*Incipient* Differential Calculus
The "Three-body Problem" 1. Three celestial bodies, with different masses, related by Newton Gravitation, are described by the following fundamental equations $m_i \frac{d^2 \vec{r}_i}{dt^2} = \vec{F}_{ji} + \vec{F}_{ki}$ (2.1) $\frac{d\vec{b}_i}{dt} = 0$ (2.2), where $\vec{F}_{ji}$, $\vec{F}_{ki}$ are the gravitational forces of the bodies *j* and *k*, respectively, on the body *i* and $\vec{b}_i$ is the angular momentum (for $i = 1, 2, 3$) 2. Eqs. (2.2) correspond to 6 scalar differential equations per body , that is to an 18-order system of ordinary non-linear differential equations. It is *intrinsically unsolvable* in Classical Mechanics, because it only has 2 first order *closed form* integrals (energy and areas) Analogous consequences in Quantum Mechanics for molecules with more than 2 atoms (e.g. water)	Perfectly solvable 1'. If the Problem is reformulated in terms of incipient derivatives $m_i \frac{\tilde{d}^2 \vec{r}_i}{\tilde{d}t^2} = \vec{F}_{ji} + \vec{F}_{ki}$ (2.1') $\frac{\tilde{d}\vec{b}_i}{\tilde{d}t} = 0$ (2.2') it has (at least) a solution in *a closed form* (as *explicitly* desired by Poincaré) 2'. In addition, it is also possible to show that such a solution can be obtained at there different hierarchical levels of Ordinality, according to the initial model adopted: a) as System made up of three *distinct* bodies (see Eqs. (2.1') and (2.2')) b) as System made up of three "*binary-duet* " sub-systems c) as one sole "*ternary* " System made up of three "*binary-duet* " sub-systems Analogous consequences in Quantum Mechanics (for molecules with more than 2 atoms) lead to a possible re-interpretation of Heisemberg's Principle

As is well known, the traditional derivative of order n can be expressed by means of Faà di Bruno's formula (see Tab. 3; Oldham & Spanier, 1974, p. 37)**.** Whereas the *incipient* derivative of Order n is given by Eq. (3.2) (Giannantoni 2002, p. 175; 2007b, p. 33; see also Appendix), where the

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general solution of *any* linear differential equation with variable coefficients in terms of the *sole* Solving Kernel; ii) such a solution, in particular, is already structured in a *closed form* (according to Poincaré's definition) and can directly be transposed to *incipient derivatives* (Giannantoni 2007b, ch. 5); iii) in addition, because the Solving Kernel is generally a *function of function*, such a *transposition* can be directly obtained by means of *Faà di Bruno's formula* (ib., ch. 3); iv) this in fact, being in turn structured in a *closed form*, can directly be transposed to the derivatives of *functions of function* when the latter are expressed in incipient terms (the only difference is that, in such a case, there are no longer "partitions" and, consequently, related "sums"); v) finally, any traditional non-linear differential equation in TDC can be transformed into a linear Ordinal differential equation in IDC, with the same methodology as already shown, for example, with reference to Riccati's Equation (Gaineville 2004).
On the other hand, such a general procedure, already adopted in other papers and books (e.g., Giannantoni, 2003b, 2004a, 2004c, 2006a), is the same which enabled us to sustain the general validity of a Differential Calculus (namely IDC), which contemporaneously operates in terms of Ordinality and cardinality (see Giannantoni 2007b, ch. 3).

Table 3 - Basic differences between *Incipient* derivative and *Traditional* Derivative of order n

Traditional Differential Calculus	*Incipient* Differential Calculus
$(\frac{d}{dt})^n e^{\alpha(t)} = (\frac{d}{dt})^{n-1}[e^{\alpha(t)} \cdot \dot{\alpha}(t)] =$ (3.1)	$(\frac{\tilde{d}}{\tilde{d}\, t})^n e^{\alpha(t)} = e^{\alpha(t)} \circ [\mathring{\alpha}(t)]^{\tilde{n}}$ (3.2)
$= e^{\alpha(t)} \cdot \{(\dot{\alpha}(t))^n + \psi[\ddot{\alpha}(t), \dddot{\alpha}(t), ..., \alpha^{(n)}(t)]\}$ (Faà di Bruno's formula)	$(\frac{\tilde{d}}{\tilde{d}\, t})^n e^{\alpha(t)} = e^{\alpha(t)} \cdot [\mathring{\alpha}(t)]^n$ (3.2')

symbol $\tilde{n}$ stands for $\tilde{n} = [n, (n)]$, which contextually indicates both the Ordinal and cardinal structure of the result so obtained (see, e. g., Eqs. (13.1), (13.2), (13.3)).
Consequently, in order to make a possible comparison between the two derivatives in terms of mere *cardinal terms*, the Ordinality in Eq. (3.2) has preliminarily to be neglected, so as to "reduce" the corresponding Ordinal Information content (Giannantoni 2006a) to a simple cardinal meaning, expressed by the "reduced" Eq. (3.2').

The comparison between Faà di Bruno's formula (3.1) and Eq. (3.2') clearly shows the "drift" effect associated to the traditional derivative with respect to the more harmonious "*persistence of form*" which characterizes the incipient derivative (Giannantoni 2004b). Such a "drift" effect represents the fundamental reason for the *intrinsic unsolvability* of the Three-body Problem by means of the traditional derivatives, whereas it always presents *a closed form* solution in terms of incipient derivatives.

Relativistic Mechanics: Time *dilatation* and Space *contraction*

According to Relativistic Mechanics, *time dilatation* and *space contraction* (see Eqs. (4.1) and (4.2) in Tab. 4) have to be considered as being intrinsic properties in physical phenomena, such as, for instance, in the *muonic decay*, usually considered as being a "proof" of Relativity Theory.
The IDC, on the contrary, enabled us to show that "*time dilatation*" and "*space contraction*" are simple consequences of a *linguistic* effect, specifically due to the Differential Calculus adopted (the so-called

Table 4 - New Results in *Relativistic* Mechanics

Absolute Differential Calculus	*Incipient* Differential Calculus
Time *dilatation* - Space *contraction*	It is a *linguistic* "drift" phenomenon
$\Delta t = \frac{\Delta t'}{\sqrt{1 - V^2/c^2}}$ (4.1)	The muonic decay: $e^{\alpha(t)}$ (4.7)
$\Delta x' = \Delta x \cdot \sqrt{1 - V^2/c^2}$ (4.2)	$(\frac{d}{dt})^2 e^{\alpha(t)} = e^{\alpha(t)} \cdot \{[\dot{\alpha}(t)]^2 + \ddot{\alpha}(t)\}$ (4.8)
$(\frac{d}{dt})\, \vec{p}_{ph} = 0$ 2nd order Diff. Equation (4.3)	$(\frac{\tilde{d}}{\tilde{d}\, t})^2 e^{\alpha(t)} = e^{\alpha(t)} \cdot [\mathring{\alpha}(t)]^2$ (4.9)
$\vec{r}_{ph}(o) = \vec{r}_{ph,0}$ (4.4) $\vec{u}_{ph}(o) = c$ (4.5)	
Basic assumption $c = const$ (4.6)	The adoption of an exponential function as a solution to the problem, now becomes even more significant, because this function can adequately represent the famous *muonic decay*

Absolute Differential Calculus). The different results are synthetically compared in Tab. 4.

The reason for such a "linguistic" effect, thoroughly analyzed in (Giannantoni 2007, pp. 73-84), always resides in the difference illustrated in Tab. 3, now specifically referred to a second order differential equation (photon momentum equation (4.3), with its associated initial conditions (4.4) and (4.5)), which is solved on the basis of the *mathematical postulate* that light speed is constant (4.6).

The comparison between the different results obtained by General Relativity (Eq. (4.8)) and by IDC (Eq. (4.9)), respectively, shows that the adoption of the TDC (even in its *Generalized Absolute* form) requires the introduction of *corrective coefficients* (see Eqs. (4.1) and (4.2)) as a consequence of its *in-adequacy* from a *linguistic* point of view.

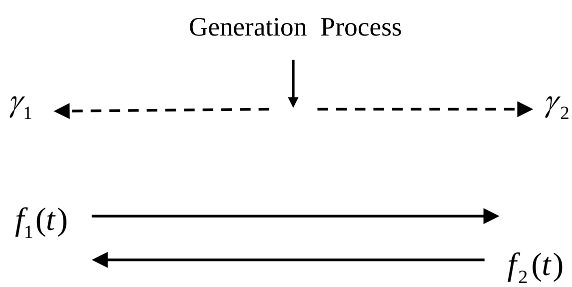
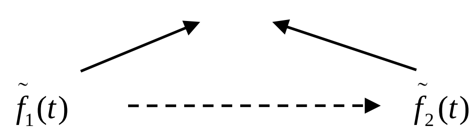
Quantum Mechanics: the "Entanglement" effect

The *entanglement* effect is the "*hinge* phenomenon in Quantum Mechanics", as anticipated by the famous physicist E. Schrödinger in 1935 (Aczel 2004, p. 49). It consists in the fact that two micro-objects (e.g. two photons), generated by the same process, remain *indissolubly related* to each other, and thus *"entangled"*. Consequently, any minimum perturbation on one of the two, *instantaneously* reflects on the other, even if they proceed in opposite directions, and even at very high distance (in principle *infinitive*).

Such a phenomenon, which is presently considered as being the most "mysterious" phenomenon in Quantum Mechanics (ib., p. xiii), when re-considered in terms of Odum's Co-production Process (see Eq. (5.1) in Tab. 6), becomes immediately explicable, under dynamic conditions, in terms of Co-production Process Ordinality and its associated cardinality (see Giannantoni 2007b, pp. 139-141).

The basic reason for such an "explicable" phenomenon resides in the fact that physicists generally research for a *direct* relationship between the two photons generated, in terms of a direct *efficient causality* between their *cardinal* physical properties and, in addition, in mere *functional* terms (see Tab. 5 and also Appendix). Whereas it is the Ordinality of the System (understood as a Whole) which "guides" the Ordinal Process: physical properties, in fact, are a simple cardinal adherent reflex of that Ordinality (ib.; see Tab. 5 and also Appendix). It is evident that such an interpretation also has important consequences on a possible re-interpretation of Heisemberg's Principle, as shown in (Giannantoni 2007b, pp. 133-145), in addition to that already mentioned in the case of the "Three-body Problem" (see related section).

Table 5 - New Results in *Quantum* Mechanics

Traditional Differential Calculus	*Incipient* Differential Calculus
The "Entanglement" effect "The *entanglement* effect is the *hinge* phenomenon in Quantum Mechanics" (E. Schrödinger, 1935)  **direct functional relationship**	Perfectly explicable in terms of Odum's Co-production $\tilde{E}\, m(u) \overset{[\to}{=} \begin{pmatrix} \tilde{E}\, m(y_1) \\ \tilde{E}\, m(y_2) \end{pmatrix}$ (5.1) **Ordinality (~)**  **cardinal physical adherent reflex**

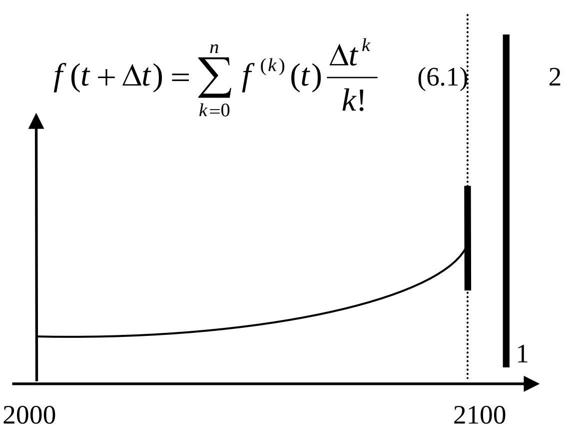
The results up to now obtained enable us to foresee some other important consequences in another extremely important field, which is much more strictly related to the themes of the Fifth Biennial Emergy Conference: *Global Warming* and associated *Climate Change*.

Global Warming and Climate Change

To illustrate both these aspects it is worth recalling that the mathematical difficulties associated to the forecast of possible future scenarios are (to such an extent) very similar to the problems concerning Weather-Forecasts. The basic difference only consists in the fact that the latter are substantially thermofluidodynamic problems (based on Stokes-Navier's Equations) whereas the former are dynamic problems related to an "Energy balance". Weather-Forecasts (see also Tab. 6), on the other hand, present intrinsic mathematical limitations which are usually synthesized by the so-called Ljapounov's Time (Strogatz 2003, p. 244-245). This "time", which is intrinsically defined by the mathematical model adopted, indicates the time interval (usually 48-72 hours) after which any solution obtained progressively loses its proper physical sense (ib.).

Mathematical problems in *Climate Change* are very similar, apart from the different time scale. So, even if we assume that *Climate Change* **(**considered as a global effect**)** really exists, we are not able to reach a definitive conclusion on the subject. In fact we are only able to depict some widely variegated "scenarios", without any possibility of foreseeing any reliable dynamic behavior in the long term (50-100 years). This is because any non-linear mathematical model based on TDC suffers from limitations accounted for in Hadamard's Theorem: in fact all the solutions are always non-linearly dependent on the initial conditions. Consequently, uncertainties about the initial conditions, associated with round-off errors, end up by "destroying" the solution very rapidly in time.
Such an aspect can be illustrated by means of a simple analogy: the evaluation of a given function $f(t)$, at the time $t+\Delta t$, through Taylor's expansion series (see Eq. (6.1) in Tab. 6).

Table 6 - *Global Warming* and *Climate Change*

Traditional Differential Calculus	*Incipient* Differential Calculus
1. Weather-Forecasts Limitations are due to Ljapounov's Time (about 48-72 hours)	1'. No limitations due to Ljapounov's Time (total absence of "drift" phenomena)
2. Climate Change (Energetic Balance) Limitations due to Hadamard's Theorem: solutions are non-linearly dependent on the initial conditions	2'. No limitations due to Hadamard's Theorem: solutions are always linearly dependent on the initial conditions
$f(t+\Delta t)=\sum_{k=0}^{n} f^{(k)}(t)\frac{\Delta t^k}{k!}$ (6.1) 	$[f^{(k)}(t)-(\frac{\tilde{d}}{\tilde{d}t})^k f(t)]=$ $=e^{\alpha(t)}\cdot\psi_k[\alpha^{(2)}(t),\alpha^{(3)}(t),...\alpha^{(k)}(t)$ (6.3)
Uncertainty $R_n(t)\propto(\frac{d}{dt})^{n+1} f(t)$ (6.2)	**a) inadequate methods adopted** **b) completely different results** **c) foreseen uncertainty even higher** **d) inadequate strategies** **e) inadequate mitigation tools** **f) inadequate judgment criteria**

In fact, the uncertainties on Global Warming trend, when this is obtained as an output of a non-linear dynamic model, can (roughly) be compared to the "uncertainty" on an analytical function. In this case, the uncertainties are proportional to the $n+1$ order derivative (see Eq. (6.2)).
Vice versa, models based on IDC no longer have those limitations related to Hadamard's Theorem, because they are always made up of Ordinal *linear* equations (see Note 7), whose corresponding solutions are always *linearly* dependent on the initial conditions (2001c, 2004b, 2004c, 2006a). This means that any desired precision can always be achieved (in the context of the solution obtained).

However, what is really worth pointing out is that *each* derivative pertaining to expansion series (6.2) presents *its own* "drift" effect with respect to the corresponding expansion series written in terms of "incipient" derivatives (see Eq. (6.3) in Tab. 6). Consequently, we can have three completely different situations according to the fact that all the single "drifts" are, respectively: i) equal to zero; ii) greater that zero; iii) lower than zero.

In the first case IDC and TDC give the same results. In the second case TDC *overestimates* an effect which is, in reality, negligible (see point 1 on the left hand side of figure in Tab. 6). In the third case, on the contrary, TDC *underestimates* an effect which could be *much higher* than the most accurate current estimations (see point 2, in the same figure).

This clearly means that our present evaluations about *Global Warming* and *Climate Change*: a) are based on the adoption of strongly inadequate methods; b) IDC, in fact, which seems to be much more adequate, leads us to completely different results; c) usual foreseen uncertainties on those important phenomena could be (in the case iii)) really much higher; d) such inadequate methods consequently lead scientists to suggest corresponding inadequate strategies to mitigate the foreseen effects; e) this also implies the adoption of inadequate mitigation tools and, at the same time, f) inappropriate judgment criteria (because based on the same original inadequate methods).

CONCLUSIONS

The examples previously shown (and their associated results) do not intend to assert that the mathematical *Language* proposed has to be considered as being the best one, in an absolute sense. The selected examples only want to show that Incipient Differential Calculus seems to be very promising in order to realize Odum's "dream": The "*Unification of Science*", when based on System Perspective, Energy hierarchy and the fundamental concept of *Emergy* (Odum 1995, pp. 365-372).

Our hope is that someone would like to collaborate in order to improve such a new Ordinal Differential Approach or, even better, to conceive a new one, hopefully much more apt to realize that general and profound intuition of Prof. H. T. Odum.

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APPENDIX. THE MEANING OF THE INCIPIENT DERIVATIVE

The meaning of the "incipient" derivative can simply be illustrated through a parallel comparison with the traditional derivative, in order to better point out the fundamental conceptual differences. This also enables us to show, at the same time, the "Com-possibility" of the two theoretical approaches.

d/dt	$\tilde{d}/\tilde{d}\,t$
a) the traditional derivative tends to describe the variations in *quantity*	a') the "incipient" derivative tends to describe the variations in *Quality* (as previously defined)
b) the traditional derivative is nothing but the formal translation of three above-mentioned	b') correspondently, the "incipient" derivative is the formal translation of the pertinent three

fundamental *pre-assumptions* (see Tab. 1) when describing physical-biological-social systems: i) *efficient causality* ii) *necessary logic* iii) *functional relationships*. These three *pre-assumptions* define an *aprioristic* perspective which evidently excludes, from its basic foundation, the possibility that any process output might ever show anything "extra", with respect to its corresponding input, as a consequence of the intrinsic (supposedly) *necessary, efficient and functional* dynamics of the system analyzed. Consequently, such a theoretical approach will never see any "output excess", exactly because it has already excluded from the very beginning (but only aprioristically) that there might be "any". In this sense it is possible to say that such an approach describes all the phenomena as they were mere "mechanisms" (see previous section on the adoption of a new concept of derivative).	fundamental *pre-assumptions* (see Tab. 1) when describing physical-biological-social systems: i) *Generative Causality*: it refers to the fact that there are Processes whose outputs always show something in "excess" with respect to their inputs. The term "generative" is simply finalized at indicating that it is worth supposing a form of "causality" which is capable of giving rise to something "extra" with respect to what it is usually foreseen (and expected) by the traditional approach; ii) *Adherent Logic*: the same concept can be referred to Logic. In fact, a different Logic is correspondently needed in order to contemplate the possibility of coming to conclusions *much richer* than their corresponding premises. This new form of Logic, in turn, can be termed as "adherent" Logic, because its conclusions are always faithfully conform to the premises. The former, however, could even be well-beyond what is strictly foreseen by the same premises when interpreted in necessary terms; iii) *Ordinal Relations*: these are an adherent consequence of both previous concepts. In fact they express that the relationships between phenomena cannot be reduced to mere *functional* relationships between the measured *cardinal* quantities, because Phenomena always "vehicle" something else, which leads us to term those relationships as "Ordinal" relationships. The term "Ordinal", that might appear as being simply adopted to make a difference with respect to its corresponding "cardinal" concept, has in reality a much more profound meaning (see later on).
c) The concept of *function* $f(t)$ is often given by means of a graph. In such a case each "point" (of the curve) represents that particular value of the function registered in correspondence to the generic time t considered. The various points, in fact, are supposed to be corresponding to physical values of a process and thus they are thought of as being "functionally" related to each other as a consequence of a subjacent *efficient cause*, which, in turn, is supposed to be the true origin of such a registered trend. Consequently, in the research for the *geometric* properties of the curve (such, as for instance, the local slope), one supposes to possibly capture the corresponding *physical law*, of a *necessary nature*, which *governs* that specific trend.	c') The concept of *Ordinal Relation* cannot be represented graphically, because graphs can only be understood (at most) as a "cipher" of an Ordinal Relation. In this case, in fact, the interpretation starts from radically different presuppositions. The single "points", although still understood as corresponding to registered values of a given phenomenon, are not thought of as "directly" related to each other as a consequence of necessary "mechanism". This is because they are no longer understood as the "result" of a "causal efficient" process, but as the *exit* (each one by itself considered) of a *one sole* Generative Activity. They correspond, in fact, to subsequent *acts of generation*, due to *the same* subjacent Generative Cause, but, at the same time, they are though of as *cardinally independent* from each other. Consequently, they are considered as

In such a sense the derivative $\dot{\alpha}(t)$ is exactly understood as a (necessary) expression of a such a *direct* relation, of a *causal efficient* nature, between the values of the function $f(t)$ at each time considered (see also later on).	being related only *indirectly*, as a simple cardinal reflex of the same Generative Activity. This is also the reason for the different notation adopted (see $\overset{\circ}{\alpha}(t)$ later on). Such an aspect, which is *ever-present* in nature, in some cases becomes particular evident (think of the Entanglement of photons)
In this case "time" (t) is understood as a physical parameter "external" to the system, according to which the sequence of events is "ordered"	Here, on the contrary, "time" (t) is understood as a physical parameter by means of which we register the Order of "dis-equilibrium" acts "internal" to the System. In this sense, each time is a "generating time". Consequently, the so-called "initial" time is no longer "conventional", because "each time is really a new beginning" (see Giannantoni 2002, p.128)
d) the definition of the traditional derivative is usually given through the *operator* Δ, defined as $$\Delta f = f(t+\Delta t) - f(t) \qquad \text{(A.1).}$$ The same term "operator" reveals the concept of *efficient causality* and, at the same time, the sign of equality ("=") reveals that the results (Δf) of that *logical definition* is understood as a necessary consequence of the premises: $f(t+\Delta t) - f(t)$. Eq. (A.1), through the introduction of a new *operator* (δ), defined as $$\delta f(t) = f(t+\Delta t) \qquad \text{(A.2),}$$ can be rewritten as follows $$\Delta f = (\delta - 1) f(t) \qquad \text{(A.3);}$$	d') The incipient derivative *of order 1* ($\tilde{d}/\tilde{d}\,t$) is defined by borrowing similar symbols from TDC, but they are interpreted in completely different way (and thus characterized by the tilde notation): i) the corresponding concept of "translation" $$\tilde{\delta}\, f(t) = f(t+\tilde{\Delta} t) \qquad \text{(A.2')}$$ is only apparently similar to Eq. (A.2). In fact, while δ (in (A.2)) defines a translation due to an *efficient* cause, the symbol $\tilde{\delta}$ indicates the simple registration of a *variation*, which is not the result of an efficient cause, but the Reflex of a Generative Causality, *both* in terms of *quantity* and *Quality* (as the tilde notation would remind); ii) The symbol $\tilde{\Delta} t$ expresses the same concept with reference to *time*: this in fact is no longer seen as a physical entity "external" to the system, but as a "cipher" of the *internal* "dis-equilibrium" of a *self-organizing* system
e) consequently, the *operator* that defines the incremental ratio can be restructured as $$\frac{\Delta}{\Delta t} = \left(\frac{\delta - 1}{\Delta t}\right) \qquad \text{(A.4)}$$	e') as an *Adherent* Consequence, the symbol $$\left(\frac{\tilde{\delta} - 1}{\tilde{\Delta} t}\right) \qquad \text{(A.4')}$$ indicates the registered variation (*both* in terms of *quantity* and *Quality*) with respect to the previous condition, when referred to the interval $\tilde{\Delta} t$

f) the concept of $\lim\limits_{\tilde{\Delta} t \to 0}$, in the definition of the traditional derivative, represents the *third* (and *last*) *operator*	f') the concept of $\tilde{Lim}\limits_{\tilde{\Delta} t:0 \to 0^+}$ is now the *Primary Indicator* of a Generative Process. It represents a sort of “window” or “threshold” (= “*Limen*” in Latin), from which we observe and describe the considered phenomenon, whereas $\tilde{\Delta} t : 0 \to 0^+$ indicates not only the initial time of our registration, but also, and more properly, the “Origin” (in its etymological sense) of something *in its specific act of being born*
g) the *dot* notation, in the first derivative of a generic function $e^{\alpha(t)}$ $\frac{d}{dt} e^{\alpha(t)} = e^{\alpha(t)} \cdot \dot{\alpha}(t)$ (A.5), synthetically indicates (as already pointed out) the result of a *necessary*, *efficient* and *functional* process	g') the “*little circle*” notation, in the first order “incipient” derivative $(\frac{\tilde{d}}{\tilde{d}\, t}) e^{\alpha(t)} = e^{\alpha(t)} \cdot \overset{\circ}{\alpha}(t)$ (A.5') synthetically indicates the *exit* (in its etymological sense, and not the “result”) of a *Generative, Adherent* and *Ordinal* Process. In this sense, $\overset{\circ}{\alpha}(t)$ represents the intimate “genetic properties” of the Process
h) the derivative of order *n* of a generic function $e^{\alpha(t)}$ (see Eq. (3.1)) $(\frac{d}{dt})^n e^{\alpha(t)} =$ $= e^{\alpha(t)} \cdot \{(\dot{\alpha}(t))^n + \psi[\ddot{\alpha}(t),....,\alpha^{(n)}(t)]\}$ is obtained by first developing, as a *primary* concept, the binomial Newton *operator* $\frac{(\delta-1)^n}{\Delta t^n} f = \frac{1}{\Delta t^n} \sum_{k=1}^{n} (-1)^k \binom{n}{k} \delta^{n-k} f(t) =$ $= \frac{1}{\Delta t^n} \sum_{k=1}^{n} (-1)^k \binom{n}{k} f[t+(n-k)\Delta t]$ (A.6) and, afterwards, by taking the limit of the obtained incremental ratio (a procedure which is substantially equivalent to a step-by-step process, repeated *n* times)	h') the derivative of order *n* of a generic function $e^{\alpha(t)}$, on the contrary (see Eq. (3.1), $(\frac{\tilde{d}}{\tilde{d}\, t})^n e^{\alpha(t)} = [\overset{\circ}{\alpha}(t)]^n \cdot e^{\alpha(t)}$ is obtained by remembering that the “incipient” derivative is defined according to the *reverse priority* (from left to right). This means that the symbol apparently “corresponding” to (A.6) $\left(\frac{\tilde{\tilde{\delta}} - 1}{\tilde{\Delta} t}\right)^n$ (A.6'), now indicates that the “genetic properties” of the Process (represented by [$\overset{\circ}{\alpha}(t)$]) are “amplified” to the Ordinality *n* and, in particular, they appear *as such* since the very beginning of the Process. In other terms they are *recognized* as an *immediate manifestation* of the Process and not by means of a step-by-step derivation. Think , for example, of “binary”, “duet” or “binary-duet” functions and so on;

i) The intuitive meaning of the traditional derivative is generally given with reference to the "slope" of a function. This is because the traditional derivative has a *phenomenological* meaning. This is why it can be interpreted as the "rapidity" (or "velocity" or "rate") of a given phenomenological *quantitative* variation. The same happens for the second order derivative when understood as the "velocity" of the "rate of variation" (thus "acceleration"), which can be related to the *concavity* of the curve. In all case, the variations of *any* order (think of an n-th derivative) are always registered "*a posteriori*", after an evolution of the process lasted for (n+1) intervals, each one of length Δt;	i') The intuitive meaning of the "incipient" derivative cannot be reduced to a geometrical interpretation because it does *not* represent a mere phenomenological description. In fact, still with reference to the same phenomenon, it tends to describe the *interior* Generative Causes of the same, which constitute the real "root" of that unexpected "excess", phenomenologically shown by the corresponding outputs of the System. Consequently, the incipient derivative of the first Order represents the *Generativity* of the Process, understood in *its basic meaning*. Superior Order derivatives represent the *Generativity Super-Abundance* of the Process analyzed. Their specific Ordinality thus manifests that ulterior "excess" of *Quality* (with respect to the basic Generativity) which characterizes that Process.
j) This also suggests it is worth pointing out that the two mentioned approaches *do not exclude each other*, because they are "*com-possible*". In fact, the traditional approach *is not able to exclude* (in principle) the "incipient" approach, as a consequence of the *absence* of any form of *perfect induction* (which would transform, only in such a case, the first approach in an *absolute* perspective). Being the (direct) Hypothetical-deductive Method based on "necessary" Logic (see Figure below) **Hypotheses** ↓ **Mathematical Formalization** ↓ **Conclusions** ↓ **Confirmation by experimental results** it is impossible to assert the *uniqueness* of the *inverse* process. That is: it is impossible to show that the hypotheses adopted are the *sole* hypotheses capable to explain those experimental results. This is because, in *necessary logic*, there exists, in principle, an infinity of other *possible* hypotheses capable to lead to the same conclusions.	j') The "incipient" approach , on the contrary, is not interested in showing that the traditional approach is "false" (in the Popper sense): *Firstly*, because in the majority of cases it obtains comparable cardinal results (although interpreted differently); *Secondly*, because the traditional approach has its own *inherent* criteria of falsification; *Thirdly*, and in particular, because the "incipient" approach proposes an *Ordinal* Perspective, which may lead to a solution exactly where the former fails. Such as, for instance, in the case of the famous "Three-body problem". In essence, the "incipient" approach would only like to show that: "*We can do better*". Where "better" means that the improvement has always to be shown on the basis of *experimental* results (for example, by comparing the solutions to the "Three-body problem" with the astronomical measurements obtained on a System made up, for instance, of Sun-Mercury-Venus).