Appendice 1

The Maximum Ordinality Principle
A Harmonious Dissonance

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ABSTRACT

The subtitle recalls a previous article written as a tribute to H.T. Odum's lifetime work. In this article I already pointed out that Odum's genial creativity, and in particular his famous Maximum Em-Power Principle, would "manifest its true relevance mainly in the future, and for many years to come. This is because the real and effective introduction of a renewed concept of Quality in Science, is able to transform any scientific aspect", including the same Classical Thermodynamics.

In such a perspective the Maximum Ordinality Principle is nothing but the re-proposition of the same Maximum Em-Power Principle, once deprived of any "residual" reference to traditional concepts of Classical Thermodynamics (such as Energy, Exergy and so on). Such a reformulation might thus appear, at a first glance, as being a sort of "dissonance" with respect to the previous formulations of the Maximum Em-Power Principle, both in steady state and dynamic conditions. Vice versa, in this way the real novelty of the Maximum Em-Power Principle emerges in a much clearer way, by contributing to give a more "harmonious" picture of the surrounding world.

The present reformulation, in fact, enables us to delineate the reference guide-lines for a real and complete re-foundation of Classical Thermodynamics, now properly understood as "Thermodynamics of Quality". This is because classical quantities (such as Energy, Exergy and

so on) simply represent a mere cardinal reflex of Generative Processes persistently evolving toward the Maximum Ordinality.

Such a re-foundation, on the other hand, is already implicit in the same Rules of Emergy Algebra, which represent one of the most important contributions to modern Science over the last four centuries.

INTRODUCTION

The paper substantially aims at showing that the Maximum Em-Power Principle (Odum H. T., 1994a, b, c) introduces a profound novelty with respect to Classical Thermodynamics.

Its general enunciation, given in one of the equivalent forms, states that "Every System reaches its Maximum Organization when maximizing the flow of processed Emergy, including that of its surrounding habitat". Such an enunciation is thus by itself sufficient to show that such a Principle represents something "new" with respect to the Principles of Classical Thermodynamics, especially because the physical quantity Emergy is defined in terms of a *non-conservative* Algebra.

The concept of something "new" (or rather, something "extra") with respect to Classical Thermodynamics is even better pointed out by the definition of Transformity, understood as a result of two independent Balances. The Transformity associated to each pathway (i) is in fact defined as (Brown & Herendeen, 1996)

$$Tr(y_i) = \frac{Em(y_i)}{Ex(y_i)} \qquad (1),$$

where the denominator derives from Classical Thermodynamics, whereas the numerator is obtained by means of the pertaining Rules of the well-know *non-conservative* Emergy Algebra.

The fundamental novelty of the Maximum Em-Power Principle, however, emerges when it is formulated under dynamic conditions. In this respect definition (1), which is generally valid under steady state conditions, could easily be extended by simply replacing the steady-state values with the corresponding instantaneous values

$$Tr[y_i(t)] = \frac{Em[y_i(t)]}{Ex[y_i(t)]} \qquad (2).$$

Such an approach, however, does not represent the unique possibility. If we consider in fact the Rules of Emergy Algebra pertaining to the three fundamental processes (Co-production, Inter-action, Feed-back), schematically represented in Fig. 1, we can easily recognize that the *non-conservative* algebra adopted substantially asserts that: i) "1 + 1 = 2 + something else" (in a Co-production); ii) whereas "1 times 1 = 1 + something extra", where this "extra" strictly depends on the nature of the process (Inter-action or Feed-back, respectively). In this sense Transformity may (also) be interpreted as a "cipher" of the internal self-organizing "activity" of the System (where the term "cipher" is here understood in a gnosiological sense). It would thus indicate that: *there are processes, in Nature, which cannot be considered as being pure* "*mechanisms*".

Such an assertion is equivalent to say that they are not describable in mere functional terms, because their outputs show an unexpected "excess" (with respect to their pertinent inputs). Such an "excess" can be termed as Quality (with a capital Q) exactly because it is no longer understood as a simple "property" or a "characteristic" of a given phenomenon, but as being any *emerging* "property" (from the considered process) *never ever reducible* to its phenomenological premises or to our traditional mental categories (Giannantoni, 2009; see also Anderson, 1972). This is also the reason why, according to the convention adopted for the term Quality, from now on all the fundamental terms referable to such a concept will analogously be capitalized.

It is then evident that, when transforming such a non-conservative Algebra (valid in steady-state conditions) to dynamic conditions, we end up by introducing a corresponding *non-conservative* Differential Calculus. This is precisely because the traditional derivative is not properly apt to represent such a concept of "cipher".

THE INCIPIENT DERIVATIVE

The introduction of a new concept of "derivative" is thus substantially due to the fact that the traditional derivative (*d/dt*) is nothing but the formal reflex of three fundamental pre-assumptions when describing physical-biological-social systems: i) *efficient causality*; ii) *necessary logic*; iii) *functional relationships*. Such an aprioristic perspective thus excludes, from its basic foundation, the possibility that any process output might ever show anything "extra", with respect to its corresponding input, as a consequence of the intrinsic (supposedly) *necessary*, *efficient* and *functional* dynamics of the system analyzed. Consequently, such a theoretical approach will never see any "output excess", exactly because it has already excluded from the very beginning (but only aprioristically) that there might be "any". In this sense it is possible to say that such an approach describes all the phenomena as they were mere "mechanisms".

Co-production, Inter-action and Feed-back Processes, on the contrary, suggest we think of a different form of "causality", precisely because their outputs always show something in "excess" with respect to their inputs. This "causality" may be termed as "generative" causality or "spring" causality or whatsoever. In all cases the basic concept is rather clear. Any term adopted is simply finalized at indicating that it is worth supposing a form of "causality" which is capable of giving rise to something "extra" with respect to what it is usually foreseen (and expected) by the traditional approach.

The same happens for Logic. In fact, a different Logic is correspondently needed in order to contemplate the possibility of coming to conclusions much richer than their corresponding premises. This new form of Logic, in turn, could correspondently be termed as "adherent" Logic, because its conclusions are always faithfully conform to the premises. Nonetheless, the conclusions could even be well-beyond what is strictly foreseen by the same premises when interpreted in strictly necessary terms.

As an adherent consequence of both previous concepts, the relationships between phenomena cannot be reduced to mere “functional” relationships between the corresponding cardinal quantities. This is because they always “vehicle” something else, which leads us to term those relationships as “Ordinal” relationships. The term “Ordinal” would thus explicitly remind us that each part of the System is related to the others exclusively because, above all, it is related to the Whole or, even better, it is “ordered” to the Whole.

Consequently, the new concept of derivative is nothing but the adherent “translation”, in formal terms, of the three new gnoseological concepts: *Generative Causality*, *Adherent Logic*, *Ordinal Relationships*.

Such a new derivative was intentionally termed as “incipient” precisely because it describes the processes in their generating activity or, preferably, because it focuses on their pertinent outputs in their specific act of being born. Its mathematical definition (already presented in Giannantoni 2001a, 2002, 2004b, 2008a, 2009) is here recalled only for the sake of clarity

$$\frac{\tilde{d}^{q}}{\tilde{d}t^{q}} f(t) = \underset{\tilde{\Delta}t:0\to 0^{+}}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t} \right)^{\tilde{q}} \circ f(t) \quad (3).$$

Its structure appears as being substantially “similar” to the traditional derivative, even if it is deeply different. The adoption of the “tilde” notation would in fact indicate that the same symbols are now understood in a substantially different way. To this purpose, before illustrating the proper meaning of definition (3), it is worth noting that the traditional increment $\Delta f(t) = f(t+\Delta t) - f(t)$ can equivalently be expressed in terms of the *“operator”* δ, which represents the variation $\delta f(t) = f(t+\Delta t)$ of the analyzed property $f(t)$:

$$\Delta f(t) = f(t+\Delta t) - f(t) = (\tilde{\delta}-1)\, f(t) \quad (4).$$

Thus the ratio $\dfrac{\tilde{\delta}-1}{\tilde{\Delta}t}$ (in definition (3)) substantially replaces the traditional incremental ratio $\dfrac{\Delta}{\Delta t}$.

The symbol “tilde”, however, should remind us that its meaning is now completely different.

The comparison between the “incipient” derivative and the traditional derivative can better be illustrated by firstly pointing out that the latter corresponds to an “operative” definition, because the priority of the operators that constitute its definition is understood from right to left, that is: i) firstly the concept of function (which is assumed to be a primary concept); ii) then the incremental ratio (of the supposedly known function); iii) thirdly the operation of limit (referred to the result of the previous two steps).

The "incipient" derivative, on the contrary, is based on the *direct priority* of the order of the three elements that constitute its definition (from left to right). This is why they acquire a completely different meaning. Let us start from the symbol $\tilde{Lim}$. The etymological origin of the word can help us: "Limit" comes from the Latin word "Limen", which means a "threshold". It could be a "threshold" of a door or of a "window", from which we observe and describe the considered phenomenon. In such a case the symbol $\tilde{\Delta} t : 0 \to 0^+$ indicates not only the initial time of our registration, but also the proper "origin" (in its etymological sense) of something new which we observe (and are going to describe) in its proper act of being born. It becomes then evident that the "operator" $\tilde{\delta}$ now registers the variation of the observed property $f(t)$, not only in terms of quantity, but also, and especially, in terms of Quality (as the symbol "tilde" would expressly remind). Thus the ratio (4) indicates not only a quantitative variation in time, but both the variation in Quality and quantity. In fact, from the very beginning of any process we can recognize its specific genesis in the form of a Co-production, Inter-action, Feed-back, respectively. We can then take explicit note of this genetic property by means of a rational number as an exponent of the Ordinal Incremental Ratio: 1/2, 2, and {2/2} respectively. Consequently, when we take the incipient (or "prior") derivative of any $f(t)$, this will keep "memory" of its genetic origin because, besides its quantity, it will result as being structured according the indication of such an exponent. This is correspondently termed as Ordinality, because it precisely expresses (as already anticipated) how each part of the output is related to all the others or, better, how it is genetically Ordered to the Whole. In this way the corresponding output "functions" ("binary", "duet", and "binary-duet" functions, respectively) result as being structured in such a way as to show that "excess" of Information which cannot be accounted for by means of traditional derivatives, because it is never reducible to its sole phenomenological premises or to our traditional mental categories (Giannantoni 2004a, 2008a, 2009). In other terms, the "incipient" derivative represents the Generativity of the considered Process, that is the output "excess" (per unit time) characterized by both its Ordinality and its related cardinality. This is also the reason why the sequence of the symbols (in Eq. (3)) is interpreted as a generative inter-action (see the symbol "$\circ$") between the three considered concepts. In this way the "incipient" derivative is also able to unify (and, at the same time, to specify) the three basic Processes, now explicitly understood in terms of Quality.

The Generativity concept, in fact, is that which unifies the three Processes, whereas the pertinent Ordinality expresses the structure of the corresponding output "functions" (as "binary", "duet", and "binary-duet" functions, respectively), which are understood as a Whole (ib.).

The adoption of “incipient” derivatives, however, is not exclusively restricted to the three afore-mentioned Processes, because definition (3) is valid for *any* fractional number q. This suggests we may also adopt such a definition to model *any* complex System, by simply considering “incipient” derivatives characterized by those rational numbers (m/n) which result as being more appropriate to each specific System analyzed.

THE MAXIMUM ORDINALITY PRINCIPLE

The above-mentioned possibility offered by definition (3) enables us to reformulate the Maximum Em-Power Principle in a more general form, that is in terms of Ordinality.

The corresponding verbal enunciation then becomes: “Every System tends to Maximize its own Ordinality, including that of the surrounding habitat”. In formal terms, this can correspondently be expressed as

$$(\tilde{d}/\tilde{d}\,t)^{(m/n)}\{\tilde{r}_s\} = 0 \qquad\qquad (m/n) \rightarrow Max$$

where (m/n) is the Ordinality of the System and $\{\tilde{r}_s\}$ is the proper Space of the System (where $\tilde{r}_s$ may be considered as being a generalized version of Hamiltonian coordinates).

Such a more general formulation was thus assumed as the preferential guide to recognize the most profound physical nature of some fundamental processes, all of them analyzed as self-organizing Systems. The successful application of such a Principle to some decisively “critical points” of various Disciplines (Giannantoni 2008a), enables us to assert that, on the basis of the Maximum Ordinality Principle: i) mathematical problems traditionally considered as being “insolvable” (e.g., the famous “Three-body Problem” (Poincaré, 1889)), become “solvable”; ii) problems widely recognized as being “intractable” (e.g. Protein Folding), became “tractable” in a reasonable computation time; iii) and problems usually thought as being “soluble” with a desired “precision” (referred to the model), continue to be soluble. However, they are always characterized by a “drift” between the foreseen behavior of the system and the corresponding experimental results. A “drift” which is generally much more marked as the order of the system increases.

We want now to show that all these aspects are intrinsically due to the same adoption of the traditional derivative.

To this purpose, we will start from considering the “drift” pertaining two well-distinct examples: Mercury’s Precessions, which is a problem that admits an *analytical* solution, and the case of Global Warming, whose future projections are obtained by means of *numerical* solutions. In both cases we will point out the basic difference between the concept of “precision” (pertaining to the model) and the concept of “drift” (which is referred to the phenomenon analyzed).

These two examples will also be able to show that the same adoption of the traditional derivative represents, at the same time, the basic reason for the insolubility (in explicit or even in a closed form) of the above-mentioned differential problems, as well as for the “intractability” of several other problems.

Mercury’s Precessions

This example, already dealt with in (Giannantoni 2004b, Giannantoni & Zoli, 2009), is here synthetically recalled because it is particularly meaningful: it represents the case of a perfectly soluble Problem which, nonetheless, shows a derivative “drift” with respect to the physical phenomenon analyzed.

The initial idea of reconsidering such a problem in a different perspective originated from the subsisting difference between the derivatives of the exponential function $e^{\alpha(t)}$ obtained on the basis of the two distinct concepts of derivative (see Table 1). In this respect it is worth noting that the assumption of the exponential function as a reference function does not represent a limitation, because any function $f(t)$ can always be structured in the form

$$f(t) = e^{\ln f(t)} = e^{\alpha(t)} \tag{7}$$

Such a choice, in addition, simplifies the exposition of the basic concepts we are going to present.

As Table 1 clearly shows, the traditional derivatives present “additional” terms (from the second order on) with respect to the incipient derivatives. Such a specific “difference” suggested the possibility of re-interpreting, by means of IDC, the “failure” of Classical Mechanics in foreseeing Mercury’s Precessions, without modifying, in any form, the space-time concepts, as vice versa happens in General Relativity (Giannantoni 2004b, Giannantoni & Zoli, 2009).

The “Two-body problem”, in fact, as traditionally modeled in Classical Mechanics, is strictly equivalent to solving a second order homogeneous differential equation with variable coefficients (Landau & Lifchitz, 1969, p. 46). At the same time it is also well known that Classical Mechanics underestimates the value of Mercury’s Precessions, by foreseeing an angular anomaly of “zero”, with respect to 42.6±0.9 sec/cy, obtained by astronomical measurements (see Note 2). It was precisely this “discrepancy” which led us to think that such an effect could be directly related to the “drift” of the second order traditional derivative with respect to the corresponding second order “incipient” derivative.

Let us then consider two distinct homogeneous second order LDEs, with variable coefficients, written in the traditional derivative (d/dt) and in the incipient derivative ($\tilde{d}/\tilde{d}\,t$)

$$\frac{d^2 f(t)}{dt^2} + A(t)\frac{df(t)}{dt} + B(t) f(t) = 0 \quad (8); \qquad \frac{\tilde{d}^2 \tilde{f}(t)}{\tilde{d} t^2} + A(t)\frac{\tilde{d}\,\tilde{f}(t)}{\tilde{d}\,t} + B(t)\,\tilde{f}(t) = 0 \quad (8.1)$$

with the same well-posed initial conditions (according to Cauchy)

$$f^{(k)}(0) = f_{k,0} \quad \text{for} \quad k = 0,1 \qquad (9) \qquad\qquad \frac{\tilde{d}^k}{\tilde{d}\,t^k}\tilde{f}(0) = f_{k,0} \quad \text{for} \quad k = 0,1 \qquad (9.1).$$

If we now research for their pertinent solutions, $f(t)$ and $\tilde{f}(t)$, respectively, both supposed to be structured in the form (7), we obtain (see also Tab. 1) the following two correspondingly associated characteristic equations:

$$\{[\dot{\alpha}(t)]^2 + \ddot{\alpha}(t)\} + A(t)\cdot\dot{\alpha}(t) + B(t) = 0 \; (10); \qquad [\overset{\circ}{\alpha}(t)]^2 + A(t)\cdot\overset{\circ}{\alpha}(t) + B(t) = 0 \; (10.1).$$

Equation (10), as a consequence of the presence of the term $\ddot{\alpha}(t)$, is a second order non-linear differential equation, which is intrinsically unsolvable in finite terms and quadratures. Equation (10.1), on the contrary, is an algebraic equation in the incipient derivative $\overset{\circ}{\alpha}(t)$, with a consequential explicit solution.

Equation (10), however, can always be re-written as follows

$$[\dot{\alpha}(t)]^2 + A(t)\cdot\dot{\alpha}(t) + B(t) = -\ddot{\alpha}(t) \qquad (11).$$

Now, by taking into account that $\dot{\alpha}(t)$ and $\overset{\circ}{\alpha}(t)$ coincide from a pure cardinal point of view (although they are radically different from a *Generative* point of view (Giannantoni 2008a, 2009)), it is easy to recognize that the difference between the two second-order differential equations (10) and (10.1) is substantially due to the fact that the former is "similar" to the latter, apart from an additional forcing term, which leads to a particular integral, whose contribution is negative when $\ddot{\alpha}(t) > 0$ and positive when $\ddot{\alpha}(t) < 0$. This is (roughly) equivalent to say that: i) when the concavity of $f(t)$ is upward, the function tends to *underestimate* the incipient solution $\tilde{f}(t)$; ii) vice versa, when the concavity is downward, the function tends to *overestimate* the solution $\tilde{f}(t)$.

This explains why Classical Mechanics, based on traditional derivatives, underestimates the astronomical effect of Mercury's Precessions and, at the same time, why "Incipient" Mechanics leads to an estimation of the angular anomaly precession of 42.45 sec/cy, which represents a satisfactory agreement with the most recent available data (Giannantoni, 2004b).(2)

Such a result can easily be extended to homogeneous linear equations of *any* order *n*, as well as to non-homogeneous linear differential equation (of *any* order *n*), because in this case we simply have additional forcing terms. This theoretical approach can also be extended to both homogeneous and non-homogeneous *non-linear* differential equations, of *any* order *n*.[(3)] An extension which becomes of particular interest especially when the analysis starts from a well-known differential mathematical model.

It is also of interest, however, to consider the consequences of such a theoretical approach when the analysis starts from the sole *output* of a mathematical model, as it happens in the next example, concerning future trends of Global Warming and Climate Change.

Let us then consider Taylor's expansion series of function (7), understood as a generic output of any given mathematical model (see Giannantoni 2008a, Giannantoni & Zoli, 2009)

$$f(t_0+\Delta t)=e^{\alpha(t_0+\Delta t)}=e^{\alpha(t_0)}+e^{\alpha(t_0)}\cdot\dot{\alpha}(t_0)\cdot\frac{\Delta t}{1!}+\sum_{k=2}^{n}\left[\frac{d^k e^{\alpha(t)}}{dt^k}\right]_{t_0}\cdot\frac{\Delta t^k}{k!} \quad (12).$$

We can then assert that the corresponding "incipient" expansion series

$$\tilde{f}(t_0+\Delta t)=e^{\alpha(t_0+\Delta t)}=e^{\alpha(t_0)}+e^{\alpha(t_0)}\cdot\overset{\circ}{\alpha}(t_0)\cdot\frac{\Delta t}{1!}+e^{\alpha(t_0)}\sum_{k=2}^{n}[\overset{\circ}{\alpha}(t_0)]^k\cdot\frac{\Delta t^k}{k!} \quad (13)$$

gives a better estimation of the real trend of the physical phenomenon analyzed. This can simply be shown by considering the differences between the terms of the same order, in the expansion series (12) and (13), as shown by Eqs. (14) and (15) in Tab. 2, respectively. [(4)]

The comparison between Eq. (14) and Eq. (15) clearly shows the "drift" effect associated to the traditional derivatives (from the second order on), with respect to the more formally harmonious "incipient" derivatives. It is precisely such a derivative "drift" that which is responsible for the *intrinsic insolubility, in explicit terms and quadratures,* of any linear differential equation with variable coefficients (and, *a fortiori*, of any non-linear differential equation) of order $n \geq 2$, when they are written in terms of d/dt.[(5)] We consequently have two sole well-distinct cases, graphically illustrated in Figure 2.

In fact the difference between each term of Taylor's traditional expansion series and the corresponding term in Taylor's *incipient* series is given by

$$Err_{nlt}^{(k)}=[(\frac{d}{dt})^k f(t)-(\frac{\tilde{d}}{\tilde{d}t})^k f(t)]\cdot\frac{\Delta t^k}{k!}=\{e^{\alpha(t)}\cdot\psi_k[\ddot{\alpha}(t),\dddot{\alpha}(t),...\alpha^{(k)}(t)]\}\cdot\frac{\Delta t^k}{k!} \quad (16).$$

This equation shows that *each* derivative pertaining to expansion series (12) presents *its own* "drift" *effect,* with respect to the corresponding term of expansion series (13). Consequently, the two above-mentioned different cases correspond to the fact that the sum of all the derivative "drifts" is: i) either greater than zero; iii) or lower than zero, respectively.[(6)] We can thus assert that: in the first case, TDC *underestimates* an effect that could be *much higher* than the most accurate mathematical models (Fig. 2, a); in the second case, on the contrary, TDC *overestimates* an effect which, in reality, is *lower* than the best simulations and, thus, could even result as being *negligible* (Fig. 2, b).

On the basis of such considerations, we can now analyze the second example, represented by those outputs of mathematical models adopted by IPCC to estimate *Global Warming* and *Climate Change*.

Global Temperature Increase Over The Period 2000-2100

In this case our expectations are that the derivative "drift" is much more marked than in Mercury's Precessions, because Global Warming is not described by a simple second order differential equation, but it is usually modeled on a system of linear and non-linear differential equations, the order of which generally ranges from 50 to 100. In addition, the time interval considered (one century) is rather long.

According to IPCC, "The best estimates for projected global warming from 1990 to the end of this century range from 1.8 to 4.0 °C (likely range 1.1 to 6.4 °C) for different scenarios (relative to 1980-1999)" (CSI 012, 2008, p. 1). The maximum scenario refers to the case in which no more action is taken to limit emissions (see Figure 3).

On the basis of Eq. (16) it is then possible to show (Giannantoni & Zoli, 2009) that both trends, which gives net increases of 6.4 °C and 1.1 °C, respectively, over the period 2000-2100 (as foreseen by IPCC), underestimate the future increasing in Temperature. In fact, Taylor's Incipient Expansion Series correspondingly give: i) in the fist case, an increase of 16.4 °C (that is 150 % higher than 6.4 °C); in the second case, an increase of 3.01°C (that is 73 % higher than the foreseen value of 1.1°C).

Similar trends are also expected for future sea level rise (as already shown in (Giannantoni & Zoli, 2009)), exactly because the latter is strictly dependent on the increase in temperature.

In all cases, a validity confirmation of the adoption of "incipient" derivatives can be obtained by considering a well-known effect happened "in the past": the "unexplained" sea level rise over the period 1900-2000.

The "Unexplained" Sea Level Rise Over The Period 1900-2000

The sea level has been rising at a rate of around 1.8 mm per year (i.e. 18 cm/cy; see Figure 4). This rate is still increasing. Measurements from the period 1993-2003 indicated a mean rate of 3.1 mm/year. (IPCC, 2007).

The real trend of such an increase has been registered by means of 23 long tide gauge records, in geologically stable environments, provided by the Permanent Service for Sea Level. Theoretical estimations, on the contrary, lead to foresee a trend of 6.0 cm/cy.[(7)]

Such a discrepancy represents a sort of "enigma". In fact: "Two processes are involved: an increase of the mass of water in the oceans (the eustatic component), derived largely from the melting of ice on land, and an increase of the volume of the ocean without change in mass (the steric component), largely caused by the thermal expansion of ocean water." (Mayer & Wahr, 2002, p. 1).

The eustatic contribution of 6 cm attributed to IPCC leads to a residual rise to be explained of 12 cm to the end of the century, which cannot be accounted for by steric expansion only. (ib.). On the other hand, other potential effects do not seem to be able to explain such a difference, because they only give marginal contributions. They consequently result as being insufficient to account for the observed drift of 12 cm. (8)

The interpretation of such a difference in terms of IDC has been given in (Giannantoni & Zoli, 2009). In such a case Eq. (13) leads to a net increase of *not less than* 17.0 cm/cy (ib.).

This result shows that the (so-called) "un-explained" recent sea level rise is due more to an intrinsic limitation of the mathematical models adopted to describe physical systems (in terms of TDC) than to new (or not yet identified) causes. At the same time, such an example represents a significant validation of the method based on IDC, since the result obtained does not refer to (foreseen) *future* trends, but concerns *past* effects, already registered ad accurately measured.

CLASSICAL THERMODYNAMICS AS THE MOST GENERAL MATHEMATICAL MODEL OF ANY COMPLEX SYSTEM

Let us now consider the three well-known Principles of Classical Thermodynamics and their mathematical formulations. It is possible to assert that such a system of equations constitutes the most general Mathematical Model of *any* Complex System, because it can also be extended to the entire Universe (supposed as a closed System). However, as a consequence of the generalization of the concept of "drift" to any Complex System when modeled in TDC (see previous section devoted to Mercury's Precessions), the same Thermodynamic Principles will always present a "drift" (according to Eq. (16)) between their theoretical projections and corresponding phenomena analyzed. We can thus assert that: i) *Energy is not properly constant*; ii) *Entropy cannot be considered as being a state variable*; iii) *Its limit does not generally tend to zero when the absolute Temperature tends to zero*.

Consequently, the Three Principles should be reformulated as follows, by passing from

$$\frac{dEn}{dt} = 0 \qquad (17)$$

$$\frac{dS}{dt} = \frac{1}{T}\frac{\delta Q}{dt} \qquad (18)$$

$$\lim_{T \to 0} \Delta S = 0 \qquad (19)$$

to

$$\Delta En \neq 0 \qquad (17.1)$$

$$\Delta S \neq \oint_{+\gamma} \frac{\delta Q}{T} = 0 \qquad (18.1)$$

$$\lim_{T \to 0} \Delta S \neq 0 \qquad (19.1).$$

Such an assertion can be sustained on the basis of two different logical procedures. The first one is "internal" to the same traditional approach, because based on well-known and achieved results. The second one is strictly related to the mathematical language adopted to formulate Eqs. (17), (18), (19), when compared with the new language developed to formulate the Maximum Ordinality Principle.
For the sake of completeness and clarity the latter case will be dealt with in a specific Appendix.

With respect to the former case, a sufficiently clear indication that Energy could no longer be considered as being properly constant is related to two well-known aspects: Mercury's Precessions and non-integrable Systems. Two results which are widely recognized as being fundamental in Physics (the former is confirmed by *experimental* results, while the latter is based on the *theoretical* impossibility of a solution to the "Three-body Problem" and, *a fortiori*, to the "N-body Problem").

First Example: The Two-body Problem (Mercury's Precessions)

We have already seen that, on the basis of Classical Mechanics, if total Energy is assumed as being constant

$$En_{tot} = En_{kin}(t) + En_{pot}(t) = const \qquad (20),$$

the explicit solution gives $\Delta\varphi_{\sec} = 0$, instead of a variation of angular anomaly of about 42 sec/cy.

If, vice versa, we assume that (Landau, Mécanique, p. 57)

$$En^{*}_{tot} = En_{kin}(t) + [En_{pot}(t) + \frac{\alpha}{r^2}] = const \qquad (20.1),$$

we get an explicit solution which is conform to experimental results, but the previous value of total Energy (given by Eq. (20)) can no longer be considered as being constant. In fact

$$En_{tot} = En_{kin}(t) + En_{pot}(t) = En^{*}_{tot} - \frac{\alpha}{r^2} \neq const \qquad (20.2).$$

The additional contribution to potential Energy (α / r^2) was initially thought of as being referable to an another planet (termed as Vulcan), supposedly situated between the Sun and Mercury. However, such an additional Planet, although thoroughly researched, *has never been found* and, at the same time, perturbations due to all the other planets *are not sufficient* to explain the considered effect.

Second Example: The “Three-Body” Problem and Non-Integrable Systems

As an immediate consequence of the insolubility of the “Three-body” Problem (when extended to the “N-body Problem”) is represented by the so-called *non-integrable* Systems. In such a case: i) Energy cannot be defined as *a state variable*, because the final state of any non-integrable System is always un-known; ii) Non-integrable Systems, at the same time, have progressively shown that the condition “En = const” has to be *removed* in favor of weaker and weaker conditions, such as: *ergodyc Systems, weekly stable Systems, chaotic Systems*; iii) “The conservation of Energy is a limitation imposed to *freedom* of complex systems” (Poincaré, 1952, p. 133); iv) “Its conservation excludes *the emergent novelty* that grows out of complex interactions” (Mirowski, 2000, p. 7).

These two example are already sufficient to assert that an extremely important Principle, such as the Maximum Em-Power Principle, should not be “anchored” to Classical Thermodynamics (neither in terms of Energy or Exergy), because, according to Popper, the “falsification” of one sole presupposition of C.T. (e.g. Energy conservation) is sufficient to “falsify” the entire Theory. This is precisely because the consequences of that sole falsification reflect on the entire Theory.

This also means that Principles of Thermodynamics can still be adopted in the form (17), (18), (19) (where the symbol “=” is now clearly understood as “$\cong$”) only for those Systems characterized by low differential orders, and for time intervals comparable with the dominating time constants of the System. In all the other cases, in fact, any mathematical model based on C.T. (and, in particular, on Energy “conservation”), will show an “intrinsic drift” effect, which generally becomes much more marked as the differential order of the system increases and/or the time of analysis is largely higher than the proper time constants of the system analyzed (such as in the case of Mercury’ Precessions).

As an example, the application of the Energy conservation Principle to the entire Universe would lead to reinterpret (in this new perspective) the so-called “Dark Energy” as a “drift” (in TDC) of about 1,400 %. This is because the Maximum Ordinality Principle shows that the traditional concept of “Energy” is nothing but a cardinal reduction of chrono-topological characteristics of the “proper” Space of the System. In this case, however, any Inter-Relation Process between two (or more) Systems (*i* and *j*), modeled in Ordinal terms, does not lead to the superimposition of the corresponding “proper” Spaces. In fact, according to the Maximum Ordinality Principle, we always have

$$\{\tilde{r}_s\} \neq \{\tilde{r}_i\} + \{\tilde{r}_i\} \qquad (21)^{(9)}.$$

Such an aspect becomes particularly evident in the previous examples concerning *Global Warming* and *Climate Change*, where the considered systems are usually analyzed on the basis of a mere super-imposition of effects of Energy quantitative terms.

CONCLUSIONS

The reformulation of the Maximum Em-Power Principle as the Maximum Ordinality Principle is thus primarily finalized to bring out that the "essence" (and the "novelty") of the former is *totally independent* from Classical Thermodynamics. In this sense the Maximum Ordinality Principle can be considered as being "a harmonious dissonance", precisely because the adjective "harmonious" refers to its faithful conformity to the "essence" of Maximum Em-Power Principle, whereas the term "dissonance" refers to Classical Thermodynamics.

The Maximum Ordinality Principle, in fact, formally expressed by Eq. (6), exactly represents the reformulation of Maximum Em-Power Principle once "deprived" of any reference to Classical Thermodynamics, by always remaining, at the same, extremely faithful to the essence of Emergy concept (or rather Transformity), when the latter is understood as a *non-conservative Phy*-sical Entity.

"Com-possibility" of Approaches

Before concluding the paper, it is worth recalling a fundamental aspect, already pointed out in (Giannantoni 2008a): the "Com-possibility" of the two Approaches (synthesized in Table 3).

The Approaches here considered, based on TDC and IDC, respectively, *do not exclude each other*. This is because the former *is not able to exclude* (in principle) the "incipient" approach, because it is based on the hypothetical-deductive method (whose structure is recalled in Tab. 3) which, in turn, is substantially based logical "necessity". Therefore, as a consequence of the *absence* of any form of *perfect induction* in "necessary" logic (which would transform, only in this case, the first approach in an *absolute* perspective), it is impossible to assert the *uniqueness* of the *inverse* process. That is: it is impossible to show that the hypotheses adopted are the *sole* hypotheses which are capable to explain the considered experimental results. In other terms, in *necessary logic* there always exists, in principle, an infinity of other *possible* hypotheses capable to lead to the same conclusions.

At the same time, the new Approach here proposed does not "exclude", *in any case*, the previous one. *Firstly*, because it is not interested in showing that the traditional approach is "false" (in the Popper sense), because it recognizes that the traditional approach already has its own specific falsification criteria. On the contrary, it is much more interested in showing that physical Processes cannot faithfully be described as mere “mechanisms”, because of the ever-present Quality in all Processes, even if Quality manifests itself in different forms and modalities (Giannantoni, 2002). *Secondly*, because the traditional approach maintains its validity for physical processes described by low order differential systems and/or time intervals comparable with the dominating time constants of the process analyzed. *Thirdly*, and in particular, because the “incipient” approach proposes an *Ordinal* Perspective which may lead us to a solution exactly where the former fails. Such as, for instance, in the case of the famous “Three-body Problem”.

In essence, the “incipient” approach would only like to show that: “*We can do better*”. Obviously, by *always* sustaining such an assertion on the basis of *experimental results*.

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APPENDIX. THE INTRINSIC DRIFT OF CLASSICAL THERMODYNAMICS

This appendix is devoted to show the general validity of Eqs. (17.1), (18.1), (19.1) with respect to the more limited validity (or absence of a general validity) of Eqs. (17), (18), (19) which, nonetheless, are considered as being the mathematical formulation of the three "Principles" of Classical Thermodynamics. Such an absence of general validity will be shown on the basis of the same mathematical language adopted (TDC), because this is the direct translation of the three basic gnoseological presuppositions of the traditional approach. This leads us to the conclusion that the afore-mentioned Thermodynamic Principles cannot be considered as being generally valid, because, for their genetic origin, they *aprioristically* “filter” any form of Generativity which, vice versa, according to the Maximum Ordinality Principle, is ever-present in “Phy-sical” Phenomena.

FIRST PRINCIPLE: EQUATION (17) BECOMES EQUATION (17.1)

The general validity of Eq. (17.1) could evidently be shown by considering the derivative “drift” of the Energy function when (in analogy to the previous examples) it is supposed to be known and also structured in its most general form

$$En = En[x(t), y(t), z(t), t] \qquad (22).$$

However, its worth adopting, in this case, an even *more general* perspective (already anticipated in Giannantoni 2002, 2004b), because referable to *any* “conservation” Principle. In fact, on the basis of the Traditional Differential Calculus we have

$$F(t) = const \quad (23) \qquad \begin{array}{c}\longrightarrow \\ \longleftarrow\end{array} \qquad \frac{d}{dt} F(t) = 0 \qquad (23.1),$$

whereas, on the basis of the Incipient Differential Calculus we only have

$$F(t) = const \quad (24) \qquad \begin{array}{c}\longrightarrow \\ \not\longleftarrow\end{array} \qquad \frac{\tilde{d}}{\tilde{d}\,t} F(t) = 0 \qquad (24.1).$$

That is, condition (24.1) is *not sufficient* to assert the general validity of condition (24).

This *does not mean* that, if (24.1) holds, we *always* have $F(t) \neq const$ (24). In some cases we can still have $F(t) = const$. However, this is not true *in general* (as happens, vice versa, to condition (23), with respect to (23.1)). This result becomes particularly evident when the function $F(t)$ is a “*function of function*(*s*)”.

Let us then consider, for example, the function

$$F(t) = f(t) \frac{\tilde{d}}{\tilde{d}\,t} g(t) \qquad (25),$$

whose structure is derived from a very general definition of Energy. This in fact, on the basis of C.T., can always be defined as

$$En(t) = \frac{1}{\theta(t)} \cdot \frac{d}{dt} g(t) = \frac{1}{\theta(t)} \cdot \frac{d}{dt} \int_{t_0}^{t} Ex[x(\tau), y(\tau), z(\tau), \tau] \cdot d\tau \qquad (26),$$

where $f(t) = 1/\theta(t)$ is the inverse of the generalized Carnot coefficient (always different from zero),

while $$g(t) = \int_{t_0}^{t} Ex[x(\tau), y(\tau), z(\tau), \tau] \cdot d\tau \qquad (27)$$

expresses total Exergy, evaluated with reference to a given arbitrary initial time t_0.

Under such an assumption, condition (23.1) becomes

$$\frac{\tilde{d}}{\tilde{d}\,t} f(t) \cdot \frac{\tilde{d}}{\tilde{d}\,t} g(t) + f(t) \cdot \left(\frac{\tilde{d}}{\tilde{d}\,t}\right)^2 g(t) = 0 \qquad (28)$$

which can be re-ordered as follows

$$\left(\frac{\tilde{d}}{\tilde{d}\,t}\right)^2 g(t) + \left[\frac{1}{f(t) \cdot} \frac{\tilde{d}}{\tilde{d}\,t} f(t)\right] \cdot \frac{\tilde{d}}{\tilde{d}\,t} g(t) = 0 \qquad (29).$$

If we now assume that $$\frac{1}{f(t)} \cdot \frac{\tilde{d}}{\tilde{d}\,t} f(t) = \beta_f(t) \qquad (30),$$

where $\beta_f(t)$, for the sake of simplicity, can be considered as being a known function, Eq. (29) can be re-written as

$$\left(\frac{\tilde{d}}{\tilde{d}\,t}\right)^2 g(t) + \beta_f(t) \cdot \frac{\tilde{d}}{\tilde{d}\,t} g(t) = 0 \qquad (31).$$

Its general solution in terms of incipient derivatives is then given by

$$g(t) = C_1 + C_2 \cdot e^{-\int_0^t \beta_f(\tau) d\tau} \qquad (32),$$

while its first derivative is equal to

$$\frac{\tilde{d}}{\tilde{d}\,t} g(t) = C_2 \cdot e^{-\int_0^t \beta_f(\tau) d\tau} [-\beta_f(t)] \qquad (33).$$

Eq. (30), on the other hand, can easily be solved in explicit terms (as a first linear differential equation)

to give $$f(t) = f(0) \cdot e^{\int_0^t \beta_f(\tau) d\tau} \qquad (34).$$

Equations (33) and (34) enable us to show that, under the general hypothesis of a function $F(t)$ structured in the form (25), condition (24.1) “generates” a *function* (or rather, a *Relationship*) *which is not constant*.

SECOND PRINCIPLE: EQUATION (18) BECOMES EQUATION (18.1)

The validity of Eq. (18.1), in lieu of Eq. (18), can be shown on the basis of Taylor’s Incipient Series (12), after having recalled the mathematical definition of the contour integral which appears on the second hand of Eq. (18.1) .

Such an integral, in fact, is defined on the basis of the concept of a traditional function of two (or more) variables. Its value can be obtained on the basis of the associated linear integral

$$\Delta S = \int_{\gamma(P_1,P_2)} Xdx + Ydy + const \qquad (35),$$

where the linear differential form $Xdx + Ydy$ is supposed to be defined in a connected field and the regular curve $\gamma(P_1, P_2)$ is defined by means of two parametric equations

$$x = x(t) \quad , \quad y = y(t) \qquad \text{for } t \in [a,b] \qquad (36).$$

It is also well-known that condition $\oint_{\pm\gamma} Xdx + Ydy = 0$ (37) is sufficient to assert that linear integral (35) is independent from the pathway $\gamma(P_1, P_2)$. It is then possible to write

$$\Delta S = \int_{\gamma(P_1,P_2)} Xdx + Ydy = \int_{t_1}^{t_2} \{X[x(t), y(t)] \cdot \dot{x}(t) + Y[x(t), y(t)] \cdot \dot{y}(t)\} \cdot dt \qquad (38)$$

or, equivalently $$\Delta S = F[x(t_2), y(t_2)] - F[x(t_1), y(t_2)] \qquad (39).$$

Accordingly, for any closed curve $\gamma(P_1, P_1)$, that is when $P_2 = P_1$ and, correspondently,

$$x(t_1) = x(t_2) \quad , \quad y(t_1) = y(t_2) \qquad (40),$$

we have that $$\Delta S = F[x(t_2), y(t_2)] - F[x(t_1), y(t_1)] = 0 \qquad (41).$$

As a consequence of the fact that the linear integral in Eq. (35) is independent from the considered pathway, the function $F[x(t_2), y(t_2)]$ can also be considered as a function of the sole parameter t. We can then define a new function $G(t) = F[x(t), y(t)]$ (42). In this way the value of $G(t_2) = F[x(t_2), y(t_2)]$ can be obtained by means of Taylor’s series (12), by starting from any given initial point $t = t_1$. We consequently have

$$\Delta S = G(t_1 + \Delta t) - G(t_1) = e^{\alpha(t_1)} \cdot \dot{\alpha}(t_0) \cdot \frac{\Delta t}{1!} + \sum_{k=2}^{n} \left[\frac{d^k e^{\alpha(t)}}{dt^k} \right]_{t_1} \cdot \frac{\Delta t^k}{k!} \qquad (42),$$

which equals zero when $P_2 = P_1$.

If we now consider the corresponding Taylor's Incipient expansion series (see Eq. (13))

$$\Delta \tilde{S} = \tilde{G}(t_1 + \Delta t) - \tilde{G}(t_1) = e^{\alpha(t_1)} \cdot \overset{\circ}{\alpha}(t_1) \cdot \frac{\Delta t}{1!} + e^{\alpha(t_0)} \sum_{k=2}^{n} [\overset{\circ}{\alpha}(t_1)]^k \cdot \frac{\Delta t^k}{k!} \qquad (43)$$

and immediately after the difference

$$\Delta S - \Delta \tilde{S} = [G(t_1 + \Delta t) - G(t_1)] - [\tilde{G}(t_1 + \Delta t) - \tilde{G}(t_1)] = [G(t_1 + \Delta t) - \tilde{G}(t_1 + \Delta t)] \quad (44),$$

(in which evidently $G(t_1) = \tilde{G}(t_1)$), we can easily recognize that, when $P_2 = P_1$, the second hand of Eq. (44) is different from zero, even if, as a consequence of the Second Principle, we have that $\Delta S = 0$. This is because Eq. (44) is an expansion series in which *each term*, for $k \in [1, \infty)$, is given by the following expression (see also Eq. (16))

$$[(\frac{d}{dt})^k G(t) - (\frac{\tilde{d}}{\tilde{d}\, t})^k \tilde{G}(t)] \cdot \frac{\Delta t^k}{k!} = \{e^{\alpha(t)} \cdot \psi_k [\ddot{\alpha}(t), \ddot{\alpha}(t), \ldots \alpha^{(k)}(t)]\} \cdot \frac{\Delta t^k}{k!} \qquad (45),$$

to which we can apply the same considerations as in the case of Mercury's Precessions.

THIRD PRINCIPLE: EQUATION (19) BECOMES EQUATION (19.1)

The general validity of Eq. (19.1) (with respect to Eq. (19)) could be shown by starting from the same Taylor's Series considered in the previous section (see Eqs. (44) and (45)), now reconsidered in a different perspective, that is according to the enunciation of the Third Principle. In this case, however, we can directly start from expansion series (42) and (43), in order to show that: even if Eq. (19) holds in a context in which ΔS is considered the result of a "necessary" process, the same equation is not generally valid when the analyzed process is considered as being a Generative Process.

Eq. (43), in fact, when evaluated for $P_2 \neq P_1$, gives

$$\Delta \tilde{G}\Big|_{P_2 \neq P_1} = e^{\alpha(t_1)} \cdot \overset{\circ}{\alpha}(t_1) \cdot \frac{\Delta t}{1!} + e^{\alpha(t_0)} \sum_{k=2}^{n} [\overset{\circ}{\alpha}(t_1)]^k \cdot \frac{\Delta t^k}{k!} \qquad (46),$$

which is generally different from zero, because: i) If $\overset{\circ}{\alpha}(t_0) > 0$ the conclusion is evident; ii) when $\overset{\circ}{\alpha}(t_0) < 0$ it can also happen that, as a consequence of the alternating signs of the various terms, the series could be equal to zero for a finite number of intervals Δt_i (i =1, 2, 3……n). This, however, can never be valid in general, because this would be true only for $\Delta t \equiv 0$.

These two cases are, by themselves, already sufficient to show that Eq. (19) is not valid in general and, consequently, it cannot properly be considered as being a Thermodynamic Principle.

For the sake of completeness it is worth adding that Eq. (46) is generally different from zero even when $\overset{\circ}{\alpha}(t_1) = 0$. This is because the estimated *global error* Err_{nlt} (obtained by summing terms given by Eq. (16)) has always to be considered as "*not less than*" (this is the reason for the pedix "*nlt*"). In fact, as already anticipated, the analysis here presented has been carried out by simply comparing TDC and IDC when the latter is (preliminary) understood in sole *cardinal* terms. In such a case all proper Space variables (x, y, z) are still considered as being in-dependent from each other. An assumption which is no longer valid when describing any System in *Ordinal* terms, because those variables are always related to each other in term of multiple "binary" functions or multiple "duet" functions (or both). This is why the global error Err_{nlt} (estimated by summing terms given by Eq. (16)) is always lower (in absolute terms) than the error that would be obtained by means of IDC when the latter is properly understood in Ordinal terms. This can easily be shown by considering the explicit expression of ΔS as function of function (and not as a function of the sole parameter t), either in terms of two arbitrary state variables, such as $x = x(t)$ and $y = y(t)$, so as to have

$$\Delta S = G(t_2) - G(t_1) = F[x(t_2), y(t_2)] - F[x(t_1), y(t_1)] \qquad (47),$$

or better, by assuming that the same temperature is one of the two variables (e.g., $x(t) = T(t)$).

In the latter case the mathematical procedure is even shorter. Nonetheless both such procedures are here omitted, not only because they require much more space than the previous ones, but also, and especially, because they are, strictly speaking, absolutely inessential. Conditions i) and ii), in fact, are by themselves already sufficient to show the absence of a general validity of Eq. (19).

Notes:

(1) As is well known, the traditional derivative of order *n* can be expressed by means of Faà di Bruno's formula (see Table 1), where the sum extends to all the partitions $(P_1, P_2, ..., P_n)$ of the integer m such as: $P_1 + P_2 + ... + P_n = m$ and $P_1 + 2P_2 + 3P_3 + ... + nP_n = n$ (Oldham & Spanier, 1974, p. 37). At the same time, for an easier and faster comparison with the integer order traditional derivatives, the

Ordinality of the incipient derivatives was directly “reduced” to a simple cardinality. This evidently represents a *preliminary* approximation, which will be released toward the end of the paper. At this stage, however, it contributes to simplify the exposition of the basic concepts.

(2) Astronomical measurements give an angular anomaly precession of 42.6±0.9 sec/cy (Landau & Lifchitz, 1966, p. 373). General Relativity (GR), on the other hand, which predicts a value of 43.0 sec/cy (ib.), cannot be considered as a definitive answer to this problem, because: it does not solve the (subsequent) “Three-body Problem”. In addition, when the latter is faced in numerical terms (in the context of GR), the solutions proposed by Sundman (1912) and Wang (1990s) become even more “intractable”. In all cases GR also adopts the same presuppositions and the same formal language (TDC), although specialized to the research for invariants. What’s more, the precision attained in the case of Mercury’s Precession is strictly depending on both the second order differential system and the corrective Lorentz transformations, which, on the other hand, substantially correspond to the introduction of an “equivalent” second order “incipient” derivative (see Giannantoni, 2008a).

(3) Such an extension is substantially based on the fact that any non-linear differential equation, written in terms of incipient derivatives, can always be transformed into a linear differential equation (Giannantoni 2007a, ch. 3).

(4) The same considerations can obviously be extended to the case in which we start from a polynomial best fit of the output.

(5) This is also the basic reason for the intrinsic insolubility of the famous Three-body Problem (Poincaré, 1889) which, on the contrary, has at least a solution in a *closed form* in terms of “incipient” derivatives (Giannantoni 2007a, ch. 5; 2008a,b).

(6) The third (theoretical) condition, corresponding to the sum of all the “drifts” identically equal to zero, can never be verified over the entire interval of analysis. At most, it is verified in a finite number of well-distinct points.

(7) The estimated component causes of sea-level change during the 20th century were summarized in IPCC’s Fourth Assessment Report (IPCC, 2007).

(8) Some Author, after having confirmed that tide gauge data are correct, concludes that “there must be a continental source of 1.4 mm/yr of fresh water.” (Wikipedia, 2008, p. 8).

(9) An interesting example is given by the well-known composition of velocities in General Relativity. Such a composition in fact represents a preliminary approximation of the condition $\{\overset{\circ}{r}_s\} \neq \{\overset{\circ}{r}_i\} + \{\overset{\circ}{r}_i\}$ (21.1), because its validity is restricted to the sole case of low differential order Systems (such as in the case of interaction between two particles).

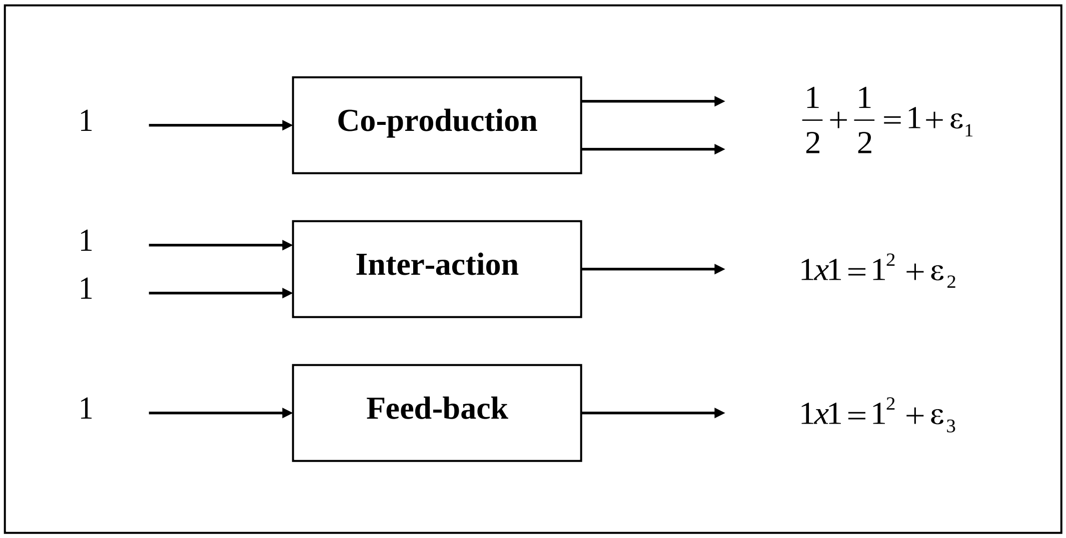
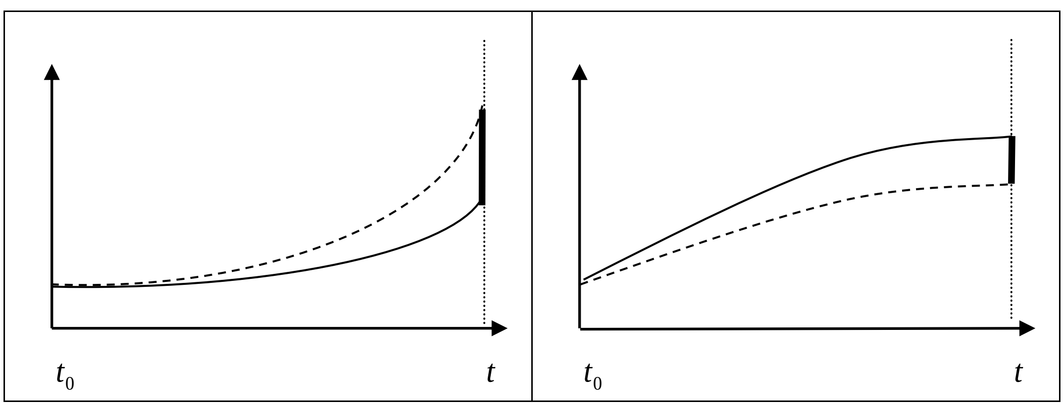


Fig. 1 - Tranformity as a “cipher” of the internal self-organizing activity of the System (*non-conservativeness* is assumed as being a fundamental aspect)



Figures 2a), b) - Over/under-estimation between Taylor’s *Traditional* and Taylor’s *Incipient* Series

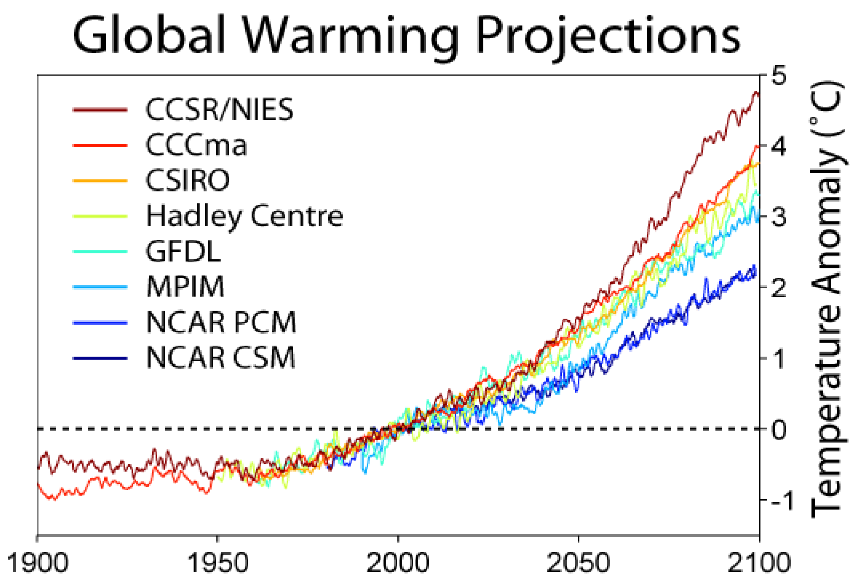


Figure 3 - Temperature Future Trends over this century according to some Research Centers - IPCC's projections are a sort of their envelope (Wikipedia, 2008a, p. 1)

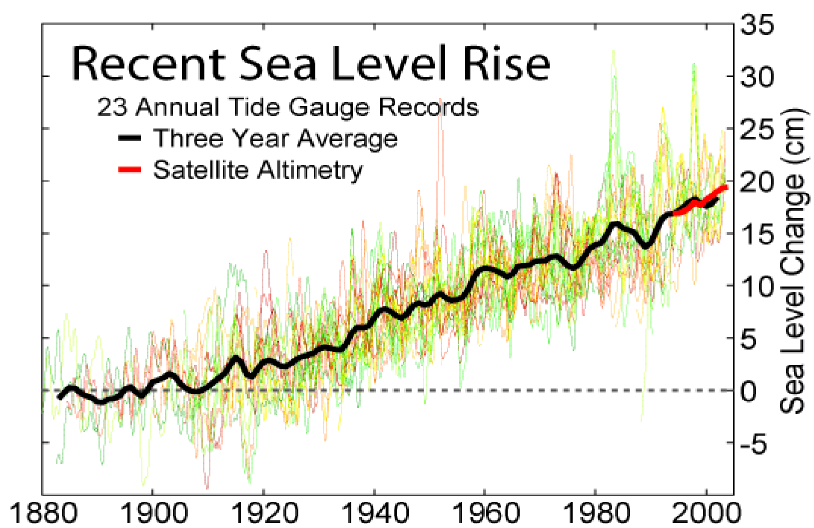


Figure 4 - Sea level rise over the period 1900-2000 (Wikipedia, 2008, p. 1)

Table 1 - Comparison between *traditional* and *incipient* derivates for the exponential function $e^{\alpha(t)}$ (1)

$\frac{de^{\alpha(t)}}{dt} = \dot{\alpha}(t)\cdot e^{\alpha(t)}$	$\frac{\tilde{d}\, e^{\alpha(t)}}{\tilde{d}\, t} = \mathring{\alpha}(t)\cdot e^{\alpha(t)}$
$\frac{d^2 e^{\alpha(t)}}{dt^2} = [\dot{\alpha}(t)]^2\cdot e^{\alpha(t)} + \ddot{\alpha}(t)\cdot e^{\alpha(t)}$	$\frac{\tilde{d}^2\, e^{\alpha(t)}}{\tilde{d}\, t^2} = [\mathring{\alpha}(t)]^2\cdot e^{\alpha(t)}$
....	....
$\frac{d^n e^{\alpha(t)}}{dt^n} = \sum \frac{e^{\alpha(t)}\cdot n!}{k_1!k_2!\ldots k_n!}\cdot\left(\frac{\dot{\alpha}}{1!}\right)^{k_1}\left(\frac{\ddot{\alpha}}{2!}\right)^{k_2}\cdot\cdot\left(\frac{\alpha^{(n)}}{n!}\right)^{k_n}$	$\frac{\tilde{d}^n\, e^{\alpha(t)}}{\tilde{d}\, t^n} = [\mathring{\alpha}(t)]^n\cdot e^{\alpha(t)}$

Table 2 - Basic differences between *incipient* derivative and *traditional* derivative of order n

Traditional Differential Calculus (Faà di Bruno's formula)	*Incipient* Differential Calculus
$(\frac{d}{dt})^n e^{\alpha(t)} = e^{\alpha(t)}\cdot[\dot{\alpha}(t)]^n +$ $+ e^{\alpha(t)}\cdot\psi[\ddot{\alpha}(t),\dddot{\alpha}(t),....,\alpha^{(n)}(t)]$ (14)	$(\frac{\tilde{d}}{\tilde{d}\, t})^n e^{\alpha(t)} = e^{\alpha(t)}\cdot[\mathring{\alpha}(t)]^n$ (15)

Table 3 - Synoptic Comparison between the Basic Presuppositions pertaining to TDC and IDC

Traditional Differential Calculus	*Incipient* Differential Calculus
1) efficient causality	1’) Generative Causality
2) necessary logic	2’) Adherent Logic
3) functional relationships	3’) Ordinal Relations

d/dt is the corresponding formal translation	$\tilde{d}/\tilde{d}t$ is the corresponding formal translation
$f(t)$ represents a *functional relationship*	$\tilde{f}(t)$ represents an *Ordinal Relationship*

<table>
<tr>
<td>Traditional approach cannot exclude the other, because of the absence of any form of perfect induction in the hypothetical-deductive method:

Structure of hypothetical-deductive method

Hypotheses
↓
Mathematical Formalization
↓
Conclusions
↓
Confirmation by experimental results</td>
<td>The new Approach does not “exclude” the former, in any case:

In fact it may only indicate (with reference to the other) that the validity of the traditional approach is limited to physical processes described by differential equations (or systems of differential equations) of very low order and/or time intervals comparable with time constants of the process analyzed</td>
</tr>
</table>