# Empowering Communities in the light of the Maximum Ordinality Principle. Selected case studies well beyond energy scarcity

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**Abstract**
*The present paper aims at showing how Empowering Communities may progressively increase their role by adopting the Maximum Ordinality Principle (MOP) as basic reference criterion. This is because its adoption on behalf of a given Community generates an increasing sense of being a real Community, together with its correlative responsibilities.*
*This aspect will preliminarily be shown in the case of adverse conditions, such as for instance energy scarcity. However, the process becomes much more intensive when the MOP is adopted to deal with aspects that go well beyond energy scarcity.*
*In such a perspective, we assumed as Reference System five European Countries (Italy, France, Spain, Germany, Austria), which were modelled as a unique and sole Self-Organizing System, that is understood as being a Real Community. In this way, after having preliminarily shown the increase of Resistance and Resilience of such an Ordinal Community adverse energy scarcity, immediately after the paper will present the most appropriate modalities for the research for new energy sources, always with reference to the same Ordinal Community. However, as a further significant contribution of the MOP, the paper will then show how the considered Countries can even more increase their consciousness of being a real and proper Community. Not only with reference to their Ordinal Relationships within themselves, but also, and especially, in terms of their Ordinal Relationships with all the other surrounding Countries, and even with the Environment.*
*All these aspects, in fact, can become an effective reality precisely because, as previously anticipated, the adoption of the MOP as Reference Principle represents a valid choice for analyzing both "non-living", "living" and "conscious" Self-Organizing Systems.*

**Keywords**: Empowering Communities, Maximum Ordinality Principle, Incipient Derivative

## 1. Introduction

The Ostensive Example adopted as an introductory case study is a System made up of five almost adjacent European Countries (Italy, France, Spain, Germany, Austria). Such countries, considered in the light of the MOP, were modelled as a unique and sole Self-Organizing System, in order to successively analyze, as already anticipated in the abstract, several and diversified aspects which go well beyond the energy scarcity.
To this purpose, it is then preliminarily worth recalling the physical and conceptual bases of the MOP and its correlative formal language.

## 2. The Reference Principle adopted for the abovementioned analysis and its correlative formal language

The Maximum Ordinality Principle (MOP), presented in 2010 at the 6th Biennial Emergy Conference, Univ. of Florida [12], is a Principle that is apt to describe the "Emerging Quality" of Self-Organizing Systems. Its verbal enunciation asserts that "*Every System tends to maximize its Ordinality, including that of its surrounding habitat*", and it is formulated by means of two fundamental equations, which are so *strictly related to each other* so as to form *a Whole* ([15], [16],[17]).

### The First Fundamental Equation

It is formulated as follows

$$(\underline{\tilde{d}/\tilde{d}t})_s^{(\tilde{m}/\tilde{n})}\{\tilde{r}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \qquad (1) \qquad (\tilde{m}/\tilde{n}) \rightarrow Max \rightarrow \{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{N}/\tilde{N}\} \qquad (1.1)$$

Where $\{\tilde{r}\}$ is the *Relational* Space of the System under consideration (see Appendix 2), while $(\tilde{m}/\tilde{n})$ represents its corresponding Ordinality, which reaches its *maximum* when it equals $\{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{N}/\tilde{N}\}$ (as indicated in Eq. (1.1)).
In this respect, it is worth noting that:

i) The underlined symbol $(\underline{\tilde{d}/\tilde{d}t})_s$ explicitly indicates that the Generative *Capacity* of the System (more appropriately termed as *Generativity*), is "*internal*" to the same System, precisely because it is the one which gives origin to its Self-Organization as a Whole;

ii) The symbol "$\overset{[\rightarrow}{=}\{\tilde{0}\}$" represents a more general version of the simple *figure* "zero", as the latter systematically appears in the traditional differential equations. In fact it now represents, at the same time:

- the specific "*origin and habitat*" *conditions* associated to the considered Ordinal Differential Equation (1);
- while the symbol "$\overset{[\rightarrow}{=}$" indicates that the System, during its Generative Evolution, is persistently "adherent" to its "origin and habitat" conditions.

**The Second Fundamental Equation**
It is formulated as follows

$$(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\{\tilde{r}\}\circledR(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}\}^{[\rightarrow}=\{\tilde{0}\} \qquad (2)$$

and it can be considered as representing a *global* Feed-Back Process of *Ordinal Nature*, which is *internal* to the same System. Equation (2), in fact, asserts that the *Relational Space* of the System $\{\tilde{r}\}$, which "emerges" as a solution from the First Equation, interacts in the form of the Relational Product ® (defined in Appendix 2) with *its proper Generative Capacity* $(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}$. In such a way as to originate a *comprehensive* Generative Capacity which, *at any time*, is always adherent to the origin and habitat conditions of the Second Fundamental Equation.

**2.1 The correlative Formal Language**
As previously shown, the MOP is formulated in terms of *a new concept of derivative*, that is, the "Incipient Derivative", whose definition is recalled in Appendix 1.
Its introduction is directly referable to the fact that Self-Organizing Systems always show an unexpected "*excess*" with respect to their phenomenological premises [29],[30],[31]. So that they usually say: "*The Whole is much more than its parts*".
Such an "excess" can be termed as *Quality* (with a capital Q) because it cannot be understood as being a simple "property" of a given phenomenon. This is because it is *never reducible* to its phenomenological premises in terms of traditional mental categories: *efficient causality*, *logical necessity*, *functional relationships*. Consequently, it cannot be described in terms of the traditional derivative that, at the level of *formal language*, represents the *perfect reflex* of such "a priori" mental categories [34].
This evidently suggests a *radically new* gnosiological perspective and, *in adherence*, the adoption of "*new mental categories*": *Emerging Causality*, *Generative Logic*, *Ordinal Relationships*.
These "*new mental categories*" can no longer be termed as "pre-suppositions", because they are not defined "a priori" (as in the case of the Traditional Approach). In fact, they are adopted "a posteriori", that is only on the basis of the "Emerging Quality" previously recognized. "*Emerging Causality*", in fact, refers to the *capacity* of a Self-Organizing System to manifest an "irreducible excess"; "*Generative Logic*" correspondently refers to the capacity of our mind to draw "*emerging conclusions*". That is "conclusions" whose information content is much higher than the information content corresponding to their logical premises, although they are persistently "adherent" to the latter. "Ordinal Relationships", in turn, refer to particular relationships of *genetic nature*, like in the case of "brothers". The latter in fact are termed as such not because of their "*direct* reciprocal relationships" (e.g. because they love each other, they respect each other, etc.), but because of their *direct reference* to *the same genetic principle*: their father (or their mother or both) [ib.].
Such new mental categories, in turn, suggest the development of a completely *new formal language*, in order to formulate *a Reference Principle*, the Maximum Ordinality Principle, so that the description of Self-Organizing Systems might result as being faithfully conform to their "Emerging Quality".
This is why *a new concept of derivative* was introduced, whose definition, as already said, is given in Appendix 1.

**3. Description of the System analyzed**
As anticipated in the Introduction, the Ostensive Example adopted as a case study is a System made up of five almost adjacent European Countries (France, Italy, Germany, Spain, Austria). Consequently, in order to analyze the problem of *energy scarcity*, at least as a simple *preliminary aspect*, the most appropriate characteristics assumed as basic reference are shown in Tab. 1.

**Tab. 1 - Basic Reference Characteristics (data from World Bank 2020)**

| Progressive Number | Nation | Imported equiv. oil per person (ton) | Imported equiv. oil per person/ Occupational level (%) | GDP ($) per person /1000/ Occupational level (%) | Occupational level (%) | GDP ($) per person |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| 1 | France | 1.8 | 2.53 | 0.570 | 71.0% | 40.494 |
| 2 | Italy | 2.0 | 3.21 | 0.533 | 62.3% | 33.228 |
| 3 | Germany | 2.4 | 3.03 | 0.586 | 79.2% | 46.445 |
| 4 | Spain | 2.5 | 3.81 | 0.452 | 65.5% | 29.600 |
| 5 | Austria | 2.5 | 3.35 | 0.734 | 75.0% | 55.050 |

From the data of Tab. 1 it is possible to recognize that, in face of a progressive incidence of imported oil on behalf of the various Nations (column 1), Italy and Spain present a higher incidence of their imported energy on the respective occupational level (column 2).
At the same time, both Italy and Spain present a lower value of GDP (per person) with respect to that of the other Nations and, in particular, when GDP is referred to the national percentage of the occupational level (column 3).

The first trend fundamentally depends on a more reduced occupational level in Italy and Spain (column 4), while the second mentioned trend principally depends on a more reduced value of the correlative GDP per person (column 5).
Now, by assuming as reference the data shown in Tab. 1, we will develop a deeper analysis, essentially based on *the first three Indicators*.
Such a choice is not simply due to the fact that the five Indicators are strictly related between them, but it is a choice mainly referable to the same *Ordinal Perspective* we are going to adopt in our analysis. In fact, with specific reference to a "Conscious" System, it is advisable to take *three* Indicators that could possibly (and adequately) represent the way of "*Thinking*, *Decision Making*, and *Acting*" on behalf of the same considered Conscious System [15]. Obviously, the three Indicators should be each time selected with specific reference to *the field of analysis* under consideration[1].
Consequently, in the specific context of *energy supply*, Indicator 1 surely represents the reference Indicator to elaborate *a general strategy* to reduce the incidence of imported energy, especially in the cases of its higher levels. Indicator 2, in turn, may suggest *the most appropriate decisions* to reduce, above all, the correlative incidence of energy supply on the occupational level. Indicator 3 may finally suggest those specific *operative actions* that could actually improve its corresponding values, especially with reference to its lower values.

**3.1 Reconfiguration of the System in Ordinal Terms, so that it might become a real "Community" of Nations**
The Reconfiguration of the System was obtained by means a Simulator, termed as EQS (Emerging Quality Simulator), based on the *explicit solution* to the MOP (shown in Appendix 2), and thus it is not conceived as a traditional computer code that operates in functional terms, but on the basis of Ordinal Relationships between the various parts of the System.
The specific Ordinal Reconfiguration of the System, modelled (as previously said) as a Self-Organizing System, was researched for in such a way as to keep substantially invariant both the minimum and maximum values of the three selected Indicators. This is because, at a preliminarily stage of the analysis, the Requalification of the System is contextually finalized to estimate the minimum *costs and times* of the same Ordinal Requalification Process.
The Ordinal properties of the System, after such an Ordinal Reconfiguration, are shown in Tab. 3, from which is possible to recognize that the minimum and maximum values of the Indicators are substantially equal to the corresponding values shown in previous Tab. 2.
In this respect, it is worth explicitly pointing out that the Ordinal Reconfiguration represented in Tab. 3 was obtained through three intermediate passages:
a) A preliminary representation of the System in terms of "couples" of Nations
b) The research for the maximally similar Reconfiguration by means of the Simulator EQS, in terms of Ordinal "Duets"
c) Finally, the disarticulation of the Ordinal "Duets" in order to obtain the Reconfiguration in terms of "single" Nations.
This procedure simply reflects the fact that in *an Ordinal context* there are not, properly speaking, "single" parts of the System, because all its various parts are related to each other *in Ordinal terms*, and the corresponding minimum level of Ordinality is precisely that represented by Ordinal "Duets".

**3.2 Requalification of the System in *operative terms***
The comparison between Tab. 2 and Tab. 3 shows that several Indicators of the real System have to be *re-qualified* in order to transform the considered System of Nations into an Ordinal Self-Organizing System or, more properly, into a real *Community of Nations*.
The Indicators that should specifically be re-qualified are shown in Tab. 4, in which:
- The values of the Indicators without any specific indication do not necessitate to be re-qualified
- The values of the Indicators that are market by an asterisk are all to be re-qualified, in order to possible assume the same values as those indicated in Tab. 3.
In particular, the symbols (+) and (-) indicate the opportunity of an increase or decrease of the corresponding values.
The values in Tab. 3 also show that even if Austria has a high value of imported oil, such an aspect is balanced by a high value of GDP per person and a rather high value of occupational percentage (see Tab. 1).
In this respect it is also worth pointing out that such a trend is substantially due to the fact that in an Ordinal Context, among the three Indicators pertaining to *each specific* Nation, there exists an Ordinal Harmony Relationship which, when expressed in terms of its *reflexed cardinalities*, asserts that

$$\text{"Indicator 1"} \times \text{"Indicator 2"} = \text{"lamda"} \times \text{"Indicator 3"} \qquad (3)$$

where "lamda" is a constant which is characteristic and specific of the field of the Ordinal System analyzed.

**3.3 The Relevance of the Ordinal Requalification Process**
At this stage one could ask: why is it worth requalifying a System of Nations in Ordinal Terms?
The answer is very simple: such an Ordinal Process, in fact, does not only increase the Resistance of the System against *adverse events* such as, for example, energy crises, as we will show *as a preliminary aspect* in the next paragraph. But it will also increase its correlative Resilience. In fact, the Ordinal Requalification Process also represents, in all cases, the Basic Reference Criterion for "Empowering Communities" in order to address all the other aspects that surely go "well beyond" energy scarcity, as we will see in the subsequent paragraphs.

[1] It is evident that the analysis can be performed with reference to an *arbitrary number* of Indictors. In such a case, however, all the various Indicators adopted as reference can be grouped into *three* distinct *classes*, which can play, in a more organic way, the same role previously described.
However, in this phase of the analysis, in which we are going to present, in particular, the fundamental characteristics of a *Methodology*, which is specific and proper of an Ordinal Analysis, for the sake of simplicity and clearness we will refer to the three Indicators previously introduced.

Without forgetting that, in addition, any considered System, by increasing its correlative Ordinality level, can adequately and progressively enhance its specific *Role*, both with respect to the Ordinal Relationships *within itself* and, even more, with respect to its surrounding *Habitat*.

**Tab. 2 - Basic Reference Characteristics of the System (taken from Tab. 1)**

| Progressive Number | Nation | Imported equiv. oil per person (ton) | Imported equiv. oil per person (ton)/ Occupational level (%) | GDP per person /1000/ Occupational level (%) |
|---|---|---|---|---|
| 1 | France | 1.8 | 2.53 | 0.570 |
| 2 | Italy | 2.0 | 3.21 | 0.533 |
| 3 | Germany | 2.4 | 3.03 | 0.586 |
| 4 | Spain | 2.5 | 3.81 | 0.452 |
| 5 | Austria | 2.5 | 3.35 | 0.734 |

**Tab. 3 - Ordinal Requalification of the System after Disarticulations of "Duets"**

| Progressive Number | Nation | Imported equiv. oil per person (ton) | Imported equiv. oil per person (ton)/ Occupational level (%) | GDP per person /1000/ Occupational level (%) |
|---|---|---|---|---|
| 1 | France | 1.8 | 2.53 | 0.570 |
| 2 | Italy | 2.0344 | 3.1509 | 0.6111 |
| 3 | Germany | 2.1391 | 3.3803 | 0.6522 |
| 4 | Spain | 2.2907 | 3.4197 | 0.6933 |
| 5 | Austria | 2.5000 | 3.3500 | 0.7345 |

**Tab. 4 Ordinal Requalification of the System in *operative terms***

| Progressive Number | Nation | Imported equiv. oil per person (ton) | Imported equiv. oil per person/ Occupational level (%) | GDP per person /1000/ Occupational level (%) |
|---|---|---|---|---|
| 1 | France | 1.8 | 2.53 | 0.570 |
| 2 | Italy | 2.0 * (-) | 3.21 * (-) | 0.533 * (+) |
| 3 | Germany | 2.4 * (-) | 3.03 * (-) | 0.586 * (+) |
| 4 | Spain | 2.5 * (-) | 3.81 * (-) | 0.452 * (++) |
| 5 | Austria | 2.5 | 3.35 | 0.734 |

**4. Description of the considered *energy crisis* and its potential effects on the System of Nations**

The evaluation of the Resistance of the System was obtained by simulating by means of EQS, and in conformity to Eq. (2), the Ordinal Inter-Action between the System in its Requalified Ordinal Configuration and the Ordinal Configuration of the same System after the estimated effects due to a foreseeable energy crisis.

In adherence to the MOP, the analysis was performed by modelling both the two configurations of the System in terms of "couples" of elements (or "Duets") for the same reasons previously recalled. Consequently, the Reference Ordinal Structure of the System is now that one represented in Tab. 5, where the values of Duets are referred, by difference, to Nation 1.

As far as the *energy crisis* is concerned, this is thought as an *energy scarcity*, which may be due either to a reduction of fossil fuels production or to an increase of their prices (or both) and, as a work hypothesis, it was supposed characterized by an "incidence" of the order of 20% on the values of Indicator 1.

The incidence on the values of the other two Indicators is strictly correlative to the Ordinal Reconfiguration of the System (shown in Tab. 5). Consequently, the corresponding effects on the System, simulated by means of EQS, and represented in Tab 6, in this case show an incidence of the order of 10% on the values of Indicator 2 and an incidence of the order of 5% on the values of Indicator 3.

More precisely, while the resulting "incidence" on the values of Indicator 1 is equal to 20%, as precisely supposed by hypothesis, the correlative "incidence" on the values of Indicator 2 is equal to 9.75%, while the "incidence" on the values of Indicator 3 evenly ranges from 5.10% and to 5.16%.

**5. Formal Translation into EQS of the Inter-Action previously described**

The evolution of the process described by the two fundamental equations (1) and (2) of the MOP was represented by means of EQS through three distinct successive processes:

a) The simulation of the Ordinal Reconfiguration of the System in its original integrity, as it appears in Tab. 5, that is considered in the absence of any external effects, and specifically structured in terms of "Duet" elements, as previously said;

b) The simulation of the Inter-Action of the System with its Habitat conditions, represented by the hypothesized *energy scarcity* and its consequential reconfiguration, still in terms of "Duets", as represented in Tab. 6;

c) The simulation of the Ordinal Inter-Action between the configuration of the System under condition a) and the System under condition b). This is because such an Inter-Action is precisely that which reflects the proper meaning of Eq. (2).

In fact, the Inter-Action between the Initial System (in its original integrity) and the System after the effects of the considered *energy scarcity*, gives origin to a *New* Ordinal System, whose final configuration is represented in Tab. 7.

**Tab. 5 - Ordinal Requalification of the System considered as being made up of 4 couples of Nations**

| l | Bl | Cl | El | $\rho_{1j(t0)}$ | $\varphi_{1j(t0)}$ | $\theta_{1j(t0)}$ | $\rho^{\circ}_{1j(t0+\Delta t)}$ | $\varphi^{\circ}_{1j(t0+\Delta t)}$ | $\theta^{\circ}_{1j(t0+\Delta t)}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 3,3416 | 0,2344 | 0,6209 | 0,0411 | 0 | 0 | 0 |
| 2 | 1 | 0 | 6,2831 | 0,3391 | 0,8583 | 0,0822 | 0 | 0 | 0 |
| 3 | 1 | 0 | 9,4247 | 0,4907 | 0,8897 | 0,1233 | 0 | 0 | 0 |
| 4 | 1 | 0 | 12,5664 | 0,7100 | 0,8499 | 0,1645 | 0 | 0 | 0 |

**Captions: Bl, Cl, Dl are internal control parameters of the Simulator EQS**

**$\rho_{1j(t0)}$, $\varphi_{1j(t0)}$, $\theta_{1j(t0)}$ represent the *actual* Relational Space {now represented by Indicators 1, 2, 3} pertaining to each Duet at the initial time**

**$\rho^{\circ}_{1j(t0+\Delta t)}$, $\varphi^{\circ}_{1j(t0+\Delta t)}$, $\theta^{\circ}_{1j(t0+\Delta t)}$ are their corresponding Incipient Derivatives**

**Tab. 6 - Ordinal Requalification of the System as a consequence of the hypothesized energy scarcity**

| l | Bl | Cl | El | $\rho_{1j(t0)}$ | $\varphi_{1j(t0)}$ | $\theta_{1j(t0)}$ | $\rho^{\circ}_{1j(t0+\Delta t)}$ | $\varphi^{\circ}_{1j(t0+\Delta t)}$ | $\theta^{\circ}_{1j(t0+\Delta t)}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 3,1416 | 0,1875 | 0,5604 | 0,0390 | 0 | 0 | 0 |
| 2 | 1 | 0 | 6,2831 | 0,2713 | 0,7746 | 0,0781 | 0 | 0 | 0 |
| 3 | 1 | 0 | 9,4247 | 0,3926 | 0,8030 | 0,1172 | 0 | 0 | 0 |
| 4 | 1 | 0 | 12,5664 | 0,5680 | 0,7399 | 0,1562 | 0 | 0 | 0 |

**Captions: Bl, Cl, Dl are internal control parameters of the Simulator EQS**

**$\rho_{1j(t0)}$, $\varphi_{1j(t0)}$, $\theta_{1j(t0)}$ represent the *actual* Relational Space {now represented by Indicators 1, 2, 3} pertaining to each Duet at the initial time**

**$\rho^{\circ}_{1j(t0+\Delta t)}$, $\varphi^{\circ}_{1j(t0+\Delta t)}$, $\theta^{\circ}_{1j(t0+\Delta t)}$ are their corresponding Incipient Derivatives**

**Tab. 7 - Final Reconfiguration of the System and correlative "mitigation" of the energy scarcity effects**

| l | Bl | Cl | El | $\rho_{1j(t0)}$ | $\varphi_{1j(t0)}$ | $\theta_{1j(t0)}$ | $\rho^{\circ}_{1j(t0+\Delta t)}$ | $\varphi^{\circ}_{1j(t0+\Delta t)}$ | $\theta^{\circ}_{1j(t0+\Delta t)}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 3,1416 | 0,2109 | 0,5919 | 0,0400 | 0 | 0 | 0 |
| 2 | 1 | 0 | 6,2831 | 0,3052 | 0,8182 | 0,0801 | 0 | 0 | 0 |
| 3 | 1 | 0 | 9,4247 | 0,4416 | 0,8482 | 0,1202 | 0 | 0 | 0 |
| 4 | 1 | 0 | 1,3490 | 0,6390 | 0,7816 | 0,1603 | 0 | 0 | 0 |

**Captions: Bl, Cl, Dl are internal control parameters of the Simulator EQS**

**$\rho_{1j(t0)}$, $\varphi_{1j(t0)}$, $\theta_{1j(t0)}$ represent the *actual* Relational Space {represented by Indicators 1, 2, 3} pertaining to each Duet at the initial time**

**$\rho^{\circ}_{1j(t0+\Delta t)}$, $\varphi^{\circ}_{1j(t0+\Delta t)}$, $\theta^{\circ}_{1j(t0+\Delta t)}$ are their corresponding Incipient Derivatives**

**6. Analysis of the results of the simulation of the previous Inter-Action Processes**

The results in Tab. 7, which refer to the Final Configuration of the System as a consequence of the hypothesized energy crisis due to *energy scarcity*, show that such an Ordinal Exit is a clear manifestation of the *recovery*, on behalf of the System, of its *Internal Stability*, as explicitly foreseen by Eq. (2) of the MOP.

In fact, it is easy to recognize the corresponding *reduction* and *mitigation* of the effects due to the reduction of imported energy, with reference to all the values of the three Indicators, both with respect to their maximum and minimum values. For the sake of brevity, but also for clarity, the corresponding "mitigations" of the effects are reproduced here below in the form of percentage changes:

| | | | | |
|---|---|---|---|---|
| Maximum Value of Indicator 1: | the incidence of | -20.0 % | becomes | -10.0 % |
| of Indicator 2: | the incidence of | - 9.75% | becomes | - 4.67 % |
| of Indicator 2: | the incidence of | - 5.16 % | becomes | - 2.18 % |
| Minimum Value of Indicator 1: | the incidence of | -20.0% | becomes | - 10.10 % |
| of Indicator 2: | the incidence of | - 9.74 % | becomes | - 4.67 % |
| of Indicator 3: | the incidence of | - 5.10 % | becomes | - 2.67 % |

It is also evident that an organic picture of the results can easily be obtained through a more articulated comparison between the values in Tabs 6 and 7, respectively.

Nonetheless, the previous results unable us to surely affirm that the System manifests a higher *Resistance* with respect to the corresponding conditions characterized by a total absence of an Ordinal Requalification.

At the same time, it is also possible to recognize a correlative increase of its *Resilience*.

In fact, in the presence of a *prior* Ordinal Requalification, the System, after having mitigated the effects of the energy scarcity, still keeps an Ordinality level sufficiently high to adequately and progressively reacquire its specific *Role*, both in terms of Ordinal Relationships *within itself* and, even more, with respect to its Ordinal Relationships with its surrounding *Habitat*. As we will see in the next paragraphs.

**7. Conclusions with specific reference to the considered aspect of *energy scarcity* previously analyzed**

The conclusions of the previous analysis can be synthesized as follows:

- In view of a possible *energy scarcity* (or, more in general, energy crises), any System of Nations should provide, in advance, to improve its Ordinal Requalification, appropriately commensurate to the "foreseeable" energy crises pertaining to its specific case;
- This is because, from such an Ordinal Requalification, it will result a "*Rebound*" of its "Resistance" and at the same time, of its correlative level of "*Resilience*".

This evidently becomes even truer, for example, in the case of the European Community (made up of 27 Nations) and, even more, in the case of USA (made up of 50 States), with particular reference to their relevant specific Role in the World.

In these cases, in fact, there is a progressive increase of the corresponding *Ordinality* of the Systems, because associated to the increasing number of their States, as clearly shown by Eq. (1.1), which is formulated, of course, in Ordinal Terms.

**8. Strategic Methodology based on the MOP for analyzing some specific cases that go well beyond energy scarcity**

After having preliminarily shown the increase of Resistance and Resilience adverse energy scarcity, and the correlative increase of consciousness of the "Empowering Community" involved, since the latter aspect is directly related to its corresponding increase of Ordinality, now we will present the most appropriate modalities to take *strategic decisions* in cases that go well beyond energy scarcity. We will then start by considering the aspects concerning both *energy saving* and the research for *new energy sources*.

**8.1 Energy saving and renewable energy sources. The Smart Grids**

The two aspects mentioned in the title represent, in a certain sense, two sides of the same coin.

In fact, in accordance with the previous analysis, *energy saving* should not be realized *unilaterally* by each single Nation. This is because, in the contest of Self-Organizing System, once Requalified in Ordinal Terms, the amount of energy saving corresponding to each Nation should always be in conformity to those of all the other Nations, that is, according to the Harmony Relationships (2.5) shown in Appendix 2. Consequently, the "actions" of the various Nations should be *coordinated* between them, because always referable to a *unique and sole* Self-Organizing System. In particular, because of the correlative benefits to its GDP, but also for the associated benefits due to the reduced "vulnerability" of the System to energy scarcity.

The same concept is equally valid inside the territory of each single Nation. This is because within any single Nation it is possible to re-propose the same analysis previously shown in the case of five Nations. In this particular case, the analysis should be performed with reference to the various Regions (or Federal States) pertaining to each Nation (18 Regions for France, 20 for Italy, 16 for Germany, 17 for Spain, 9 for Austria). Consequently, also in this case the energy saving corresponding to each Region (inside the same Nation), should always be *in adherence* to those of all the other Regions (of the same Nation), according to the previously recalled Harmony Relationships.

**8.2 Smart Grids**

An analogous concept is equivalently valid even when the energy saving is obtained by means of the recourse to *Smart Grids*. In fact, said in more explicit terms, the same diffusion of the Smart Grids should be *uniform* between the various

Nations and, contextually, among their pertinent Regions, always for the respect of the above mentioned Harmony Relationships and the correlative reduction of the "vulnerability" to energy scarcity.
However, one could ask: how is it possible to realize *a wide diffusion* of Smart Grids, given their well-known *intrinsic instability and consequential frequent blackouts*?
The answer, once again, can be found on the basis of the MOP. In fact, the problem can easily be solved by modelling a Smart Grid in the light of the M.O.P. [20]. This is because, as pointed out by P. Anderson (Nobel Prize *in Physics* 1977): "*A complex aggregate of electrons shows properties that are not reducible to their sum*" [1]. In other words, "*a complex aggregate of electrons*", although "forced" by some generators into electrical circuits, always tends to behave as a "*Self-Organizing System*".
Consequently, the distribution of the *N* Generators (and their related connections) should not be designed in *functional* terms. On the contrary, they should *topologically* be distributed in such way as the Voltage ($\tilde{V}_i$), Current ($\tilde{I}_i$) and Phase ($\tilde{\Phi}_i$) of each generator satisfy, at any time *t*, the *Harmony Relationships* pertaining to the Smart Grid under consideration, according to Eq. (2.5) in Appendix 2

$$\{\tilde{V}_{1,j+1}, \tilde{I}_{1,j+1}, \tilde{\Phi}_{1,j+1}\}_t^* = (\sqrt[N-1]{\{\tilde{1}\}})_j \circledR \{\tilde{V}_{12}, \tilde{I}_{12}, \tilde{\Phi}_{12}\}_t \qquad j = 1,3,.....N-1 \qquad (4)$$

where the index "12" refers to an *arbitrary* couple of generators assumed as reference, while $(\sqrt[N-1]{\tilde{1}})$ represents the N-1 Ordinal Roots of Ordinal Unity ($\tilde{1}$).
In this way the Smart Grid will result, in actual fact, as being *intrinsically stable*, because the specific "weight" of its internal cardinalities (always understood as a Whole) will result as being much higher than the quantitative contributions (in number and intensity) of the most frequent disturbances and, especially, in the case of unforeseeable *cyber attacks.*
At the same time, this example also suggests a possible "transposition" to *a higher level* of analysis: the Economics Stability of a System of Nations.

**9. Economics Stability of a System of Nations and their increasing consciousness as being a real Community**
The research for equilibrium conditions in a free-market economy, generally in a progressive development, but also in the case of potential crises, can be obtained once again by means of a *specific strategy* based on the MOP, because in such a case the latter allows us to solve the famous *The three-good two factor Problem* of Neo-Classical Economics. The latter in fact, in spite of its wide theoretical diffusion, is characterized, from its same origin (at the beginning of the XX century) by such an unsolvable problem, which has never been solved up to now ([27], pp. 247-252).
This Problem, as its same "title" clearly states, consists in the fact that, given three goods, in a free market economy, characterized by two productive factors (*Kapital* and *Labour*), such three goods do not reach an equilibrium condition.
This result clearly shows that a free market economy cannot be considered as being a simple "mechanism".
A free market, in fact, as shown in [20], is characterized by "Initiative", "Inventiveness" (understood as a "continuous development of new products"), without considering that *any transaction* always generates "*Extra*" *Benefits of Ordinal Nature* [11], which are *irreducible* to a traditional description in terms of *causality, necessity, functionality*.
All these conditions suggest that a free market between Nations can be more appropriately modelled as a "Self-Organizing System". In fact, when "*The three-good two factor Problem*" is interpreted in the light of the MOP, the Problem can be solved for an *arbitrary number of goods* ($N_g$), in the presence of *Three Productive Factors: Capital (K), Labour (L) and Natural Resources (N)* [14],[20].
In particular, in the case of a System of Nations, the *number of goods* ($N_g$) is precisely represented by the *number of Economies* of the considered Nations, while *Capital (K) stands for GDP, Labour (L) for occupied workers, while Natural Resources (N) refer to both internal and external resources pertaining to each considered Nation.*
As widely shown in [20], the corresponding "Emerging Solution" is given by the following *Harmony Relationships* (ib.)

$$\{\tilde{K}_{1,j+1}, \tilde{L}_{1,j+1}, \tilde{N}_{1,j+1}\}^* = (\sqrt[N-1]{\{\tilde{1}\}})_j \circledR \{\tilde{K}_{12}, \tilde{L}_{12}, \tilde{N}_{12}\} \qquad j = 1,3,.....N_g - 1 \qquad (5),$$

where the index "12", represents an *arbitrary* couple of Nations assumed as a reference, while the term $(\sqrt[N-1]{\{\tilde{1}\}})_j$ represents the $N_g - 1$ Ordinal Roots of Ordinal Unity $\{\tilde{1}\}$ (see also Appendix 2).
On the basis of such a result, a System of Nations that adopts an Economics Policy in conformity to the previous *strategic solution*, will surely increase its sense of consciousness of being a real Community. This is particularly important not only for the System of five Nations previously considered, but even more, for example, in the case of the European "Community", made up of 27 States (or Nations).

**10. Increase of Ordinality and Consciousness of a "Community" both in its Internal and External Relationships**
The examples previous considered also suggest an even *higher level* of analysis for *strategic decisions*, always based on the MOP, with particular reference to the *surrounding Habitat*.
In fact, the verbal enunciation of the MOP asserts that:

"*Every System tends to maximize its Ordinality, including* that of *its surrounding habitat*".

This means that, if for example the European Community has reached its maximum level of Ordinality, characterized by its corresponding internal Stability, and consequently it has reached a much deeper consciousness of being an effective Community, all these aspects can have a direct reflex on *the improvement* of its Ordinal Relationships. Both internal to the single Nations and between them. In addition, a further improved level of Relationships can manifest when such Communities will establish New *Ordinal Relationships* with other countries.
For example, let us consider the European Community. It may further increase its already reached level of Ordinality because it can surely play, always on the basis of the MOP, a particular "leading role" with respect to those Nations that, for example, would like to become part of the same European Community and which, at present, represent only its *Habitat*.
Something similar, or even more significantly, can be asserted with reference to United States of America (made up of 50 States), especially for their extremely important role and their relevant "influence" at Mundial Level.
The abovementioned aspects also suggest, on the other hand, that any considered Ordinal System of Nations could also play a decisive role in *the respect of the Environment*, by assuming strategic decisions always in the light of the MOP.

**11. Environment. Climate change forecasts. The Sea Level Rise over the Period 1900-2000**
In this case, the "Empowering Communities" may manifest their increase of consciousness and their particular role with reference to the Environment, only in the respect, however, of the following conditions:
a) The "prior condition" is that they have reached a sufficiently high level of Ordinality as Self-Organizing Systems, by adopting as a Reference "Guide" the Maximum Ordinality Principle;
b) Afterwards, they know very well the Environmental Phenomenology pertaining to the aspects of specific interest;
c) This means that they are able to recognize the "*Emerging Quality*" of the processes they are going to deal with;
d) And, as a fundamental aspect, they are systematically oriented at the research for a possible Syntony with such an "*Emerging Quality*";
e) Finally, even in the case of potential adverse events, they can mitigate the associated undesired "effects", always in adherence to the MOP.
In order to illustrate the importance of the previous conditions, let us consider, as an ostensive example, the Sea Level Rise over the Period 1900-2000, as described in [22], [33].
It is evident that such a process is difficult to contrast if its "origin" is not deeply known.
At a first glance, in fact, the process seems to be inexplicable, because the correlative "causes" are still unknown [26].
However, as clearly shown in [20], this is simply due to the fact that the specific "causes" are systematically researched for in terms of *efficient causality*, *logical necessity*, *functional relationships*, that is they are researched for as the various processes involved were pure "mechanisms" (as illustrated at par. 2.1).
In reality the process of Sea Level Rise can be analyzed in the light of the M.O.P. by means of its associated Ordinal Simulator EQS (ib.), which faithfully represents the various Harmony Relationships between *all* the *different* physical Systems involved in the process (sea, ice, hearth, sun, etc.). Such Inter-Actions in fact, because of their *Ordinal Nature*, are precisely those that represent the real "*generative cause*" of that registered "unexpected" trend. Which, according to such an interpretation, is nothing but an "*Emerging Exit*" of *a unique* "*Self-Organizing System*".
Consequently, "Empowering Communities" should correspondently modify their way of "*Thinking*, *Decision Making*, and *Acting*" so as to research for the maximum Syntony (and possibly Harmony) with the "Emerging Quality" shown by the considered Processes, so as to minimize both present and future effects with respect to the Environment [13].

**12. Conclusions**
We have shown how various forms of Communities, although substantially different among them for their Ordinality Level, may progressively become more intensive "Empowering Communities" by adopting as reference criterion the Maximum Ordinality Principle (MOP).
This is because its adoption, on behalf of a given Community, generates an increasing *sense of Community* and a higher level of correlative responsibility, as it has been preliminarily shown with reference to adverse conditions associated to energy scarcity. However, the same process becomes even more intensive when the MOP is adopted to deal with aspects that go well beyond energy scarcity.
In such a perspective, we have preliminarily assumed as Reference "Community" that one made up of five European Countries (France, Italy, Germany, Spain, Austria) which were modelled as a *unique and sole* Self-Organizing System, so that it could be considered as being as a *real* Community. In this way, after having preliminarily shown the increase of Resistance and Resilience of such an Ordinal Community adverse energy scarcity, immediately after we showed the most appropriate modalities for the same Ordinal Community to research for new energy sources.
As a further and more significant contribution, we also showed how the considered Countries can even increase their consciousness of being a real Community, especially with reference to their Ordinal Relationships with all the other surrounding Countries, in particular, in the case of European Community and the United States of America.
In such a general context, we have also shown how such progressively "Empowering Communities" can also improve their Ordinal Relationships with the Environment.
All these aspects, in fact, can always be realized on the basis of the Maximum Ordinality Principle, when the latter is adopted on behalf of any Conscious System in all the various cases of interest, as a preferential "Guide" for its way of "*Thinking*, *Decision Making*, and *Acting*"[15].
In this way any Community, understood as Self-Organizing *Conscious* System, can progressively maximize its proper Ordinality, so as to reach the maximum Syntony (and possibly Harmony) with the "emerging Quality" which is specific of any surrounding Habitat, included the same Environment.
In particular, this becomes possible because the MOP precisely represents a valid Reference Principle for analyzing both "non-living", "living" and "conscious" Self-Organizing Systems [16].

**Appendix 1. Definition of the "*Incipient Derivative*"**
The *Incipient Derivative* is defined as follows

$$(\frac{\tilde{d}}{\tilde{d}t})^{\tilde{q}} f(t) = \underset{\tilde{\Delta}t:0\to 0^+}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t}\right)^{\tilde{q}} \circ f(t) \qquad \text{for} \quad \tilde{q} = \tilde{m}/\tilde{n} \qquad (1.1).$$

A definition which clearly shows that the "*Incipient Derivative*" is not an "operator", like the derivative $(d/dt)$ in the Traditional Differential Calculus (TDC), but it could be termed as a "*generator*", because it describes a Process *in its same act of being born* [3],[7]. In fact:
i) The sequence of the symbols is now interpreted according to the *direct priority* of the order of the three elements that constitute its definition (that is from left to right). This is why they acquire a completely different meaning with respect to the traditional one;
ii) The three symbols, in fact, do not represent "three" distinct operations, but a *unique and sole* Generative Process;
iii) The symbol $\tilde{Lim}$, whose etymological origin comes from the Latin word "Limen" (which means a "threshold"), represents the "threshold" of that "ideal window" from which we observe and describe the considered phenomenon;
iv) The symbol $\tilde{\Delta}t : 0 \to 0^+$ now indicates not only the initial time of our registration, but also the proper "*origin*" (in its etymological sense) of something new which we observe (and describe) in its proper act of being born;
v) It is then evident that the "operator" $\tilde{\delta}$ now registers the variation of the observed property $f(t)$, not only in terms of quantity, but also, and especially, in terms of Quality (as the symbol "tilde" would expressly remind). Thus the ratio which appears in Eq. (3) indicates not only a quantitative variation in time, but both the variation in Quality and quantity;
vi) Consequently, when we take the incipient (or "prior") derivative of Ordinality $\tilde{q}$ of any $f(t)$, the *exit* of such a process will keep "memory" of its genetic origin because, besides its quantity, it will result as being structured according the indication of such an exponent. The latter in fact is properly termed as *Ordinality*, because it precisely expresses how each part of the output is *genetically Ordered* to the Whole and, at the same time, how each part is related to all the others in terms of *Ordinal Relationships* (as shown by Eqs. (A1) and (A5) in Appendix 2);
vii) In this way the "incipient" derivative represents the *Generativity* of the considered Process, that is the output "excess" (per unit time) characterized by both its Ordinality and its related cardinality, while the sequence of the symbols in its definition (Eq. (3)) can be interpreted as representing a *unique inter-action process* between the same;
viii) The above-mentioned reasons clearly show why the "incipient" derivative is able to *unify* (and, at the same time, to specify) the description of the various Self-Organizing Processes, when explicitly understood in terms of Quality;
ix) This also means that the Incipient Derivative has an *exit* that is generally different from the *result* of the corresponding derivative in TDC, even when its Ordinality is reduced to *a mere cardinality*. For example, the derivative of order *n* of the function $e^{\alpha(t)}$ (evaluated according to Faà di Bruno's formula) and the corresponding incipient derivative, give

$$(\frac{d}{dt})^n e^{\alpha(t)} = e^{\alpha(t)} \sum \frac{n!}{k_1!k_2!\ldots k_n!} \cdot \left(\frac{\dot{\alpha}}{1!}\right)^{k_1} \left(\frac{\ddot{\alpha}}{2!}\right)^{k_2} \cdot\cdot \left(\frac{\alpha^{(n)}}{n!}\right)^{k_n} \quad (1.2) \quad \text{and} \quad (\frac{\tilde{d}}{\tilde{d}t})^n e^{\alpha(t)} \overset{*}{=} e^{\alpha(t)} \cdot [\overset{\circ}{\alpha}(t)]^n \quad (1.3)$$

respectively, where $\overset{\circ}{\alpha}(t)$ represents the first order incipient derivative. And even if in some cases they coincide (for instance when $\alpha(t)$ is linear) such a coincidence has to be seen in the light of the symbol $\overset{*}{=}$ in Eq. (1.3), which reminds us that any incipient derivative is always the *exit* of a of a *generative* logical process and not of a *necessary* logical process.

**Appendix 2. General Explicit Solution to the two Fundamental Equations of the MOP, understood as a Whole** [19]
As an introduction, it is worth mentioning that precisely such a solution was the one that enabled us to solve the famous "*The three-good two factor Problem*" of Neo-Classical Economics, previously mentioned at par. 9.
Consequently, such a solution also enabled us to analyze the System of five Nations (par. 3 on) as a Self-Organizing System in the light of the MOP.
The first Fundamental Equation of the MOP (Eq. (1)), in fact, precisely because formulated in terms of Incipient Derivatives, always presents an *explicit solution*, which can always be written in the general form

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\left\{\begin{pmatrix}\tilde{\alpha}_{11}(t)\\ \tilde{\alpha}_{21}(t)\\ \ldots\ldots\\ \tilde{\alpha}_{N1}(t)\end{pmatrix}, \begin{pmatrix}\tilde{\alpha}_{12}(t)\\ \tilde{\alpha}_{22}(t)\\ \ldots\ldots\\ \tilde{\alpha}_{N2}(t)\end{pmatrix} \begin{pmatrix}\ldots\ldots\\ \ldots\ldots\\ \ldots\ldots\\ \ldots\ldots\end{pmatrix} \begin{pmatrix}\tilde{\alpha}_{1N}(t)\\ \tilde{\alpha}_{2N}(t)\\ \ldots\ldots\\ \tilde{\alpha}_{NN}(t)\end{pmatrix}\right\}} \qquad (2.1)$$

where the *Relational Space* $\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}}$ depends on the Nature of the System analyzed, while the corresponding structure of each term of the Ordinal Matrix depends on the Specific Generativity $(\tilde{d}/\tilde{d}t)_s$.

For example, if the *Relational Space* of the System is represented by $\{\tilde{\rho},\ \tilde{\varphi},\ \tilde{\vartheta}\}$, that is three topological coordinates always considered as *the exit of a Generative Process*, we can assume

$$\{\tilde{r}\}_s = e^{\tilde{\alpha}(t)} = e^{\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\}} \tag{2.2}$$

This is because, on the basis of a generalized form of De Moivre representation, it is always possible to write

$$\{\tilde{r}\}_s = \{\tilde{\rho}\circledR\tilde{i}\circledR e^{\tilde{\varphi}\circledR\tilde{j}}\circledR e^{\tilde{\vartheta}\circledR\tilde{k}}\} = \{e^{\tilde{\sigma}\circledR\tilde{i}}\circledR e^{\tilde{\varphi}\circledR\tilde{j}}\circledR e^{\tilde{\vartheta}\circledR\tilde{k}}\} = e^{\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\}} = e^{\tilde{\alpha}(t)} \tag{2.3}$$

where the traditional versors $\vec{i}$ , $\vec{j}$ , $\vec{k}$ are now replaced by three unit *spinors* $\tilde{i},\ \tilde{j},\ \tilde{k}$, which are defined in such a way as to satisfy the following *Relational Product* Rules:

$$\tilde{i}\circledR\tilde{i} = \oplus 1 \qquad \tilde{i}\circledR\tilde{j} = \tilde{j} \qquad \tilde{i}\circledR\tilde{k} = \tilde{k} \tag{2.4.1}$$

$$\tilde{j}\circledR\tilde{i} = \tilde{j} \qquad \tilde{j}\circledR\tilde{j} = \Theta 1 \qquad \tilde{j}\circledR\tilde{k} = \tilde{k} \tag{2.4.2}$$

$$\tilde{k}\circledR\tilde{i} = \tilde{k} \qquad \tilde{k}\circledR\tilde{j} = \tilde{k} \qquad \tilde{k}\circledR\tilde{k} = \Theta 1 \tag{2.4.3},$$

where the symbols $\oplus$ and $\circledR$ express more intimate relationships between the same: both in terms of sum ( $\oplus$ ) and in terms of (relational) product ($\circledR$) with respect to the case of traditional versors $\vec{i}$ , $\vec{j}$ , $\vec{k}$ .
So that representation (A3) is similar (albeit not strictly equivalent) to a system of three complex numbers, characterized by one real unit $(\tilde{i})$ and two imaginary units ( $\tilde{j}$ and $\tilde{k}$ ).
Equation (A1) thus describes the *Generative Evolution* of the System as the exit of an *Ordinal Cooperation* of *N Co-Productions* (vertical brackets) and their associated *N Inter-Actions* (horizontal brackets). At the same time, when the Process has reached its Maximum Ordinality, each term $\tilde{\alpha}_{ij}(t)$ of the Ordinal Matrix, as a consequence of such a Maximization Process, will transform into a binary-duet Relationship, represented as $\{\tilde{\alpha}_{ij}(t)\}^{\{\tilde{2}/\tilde{2}\}}$.
At the same time, the adoption of the brackets "{}" in Eq. (A1) is explicitly finalized to remind us that the Ordinal Matrix represents the Ordinal Structure of the System understood as a *Whole*.
In fact, all the elements of the Ordinal Matrix (in Eq. (A1)) satisfy the following "Ordinal Relationships"

$$\{\tilde{\alpha}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} (\sqrt[N-1]{\{\tilde{1}\}})_j \circledR \{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\} \quad \text{for } j=1,2,3,\ldots.N-1 \tag{2.5},$$

where the additional terms $\{\tilde{\lambda}_{i,j}(t)\}^{\{\tilde{2}/\tilde{2}\}}$ explicitly account for the associated habitat conditions.
Eqs. (A5) can also be termed as "Harmony Relationships" precisely because they show that all the elements $\{\tilde{\alpha}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$ of the Ordinal Matrix can be obtained by means of *one sole* arbitrary couple (Duet) $\{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$, assumed as reference, and the N-1 Ordinal Roots $(\sqrt[N-1]{\{\tilde{1}\}})_j$ of Unity $\{\tilde{1}\}$.
Consequently, if each element of the Ordinal Matrix (in Eq. (A1)) is expressed in terms of the reference couple $\{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$, the solution to the first Fundamental Equation (1) assumes the form

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}\circ\left\{\begin{pmatrix}(\sqrt[N-1]{\{\tilde{1}\}})_{11}\\(\sqrt[N-1]{\{\tilde{1}\}})_{21}\\\cdots\cdots\\(\sqrt[N-1]{\{\tilde{1}\}})_{N1}\end{pmatrix},\begin{pmatrix}(\sqrt[N-1]{\{\tilde{1}\}})_{12}\\(\sqrt[N-1]{\{\tilde{1}\}})_{22}\\\cdots\cdots\\(\sqrt[N-1]{\{\tilde{1}\}})_{N2}\end{pmatrix}\begin{pmatrix}\cdots\cdots\\\cdots\cdots\\\cdots\cdots\\\cdots\cdots\end{pmatrix}\begin{pmatrix}(\sqrt[N-1]{\{\tilde{1}\}})_{1N}\\(\sqrt[N-1]{\{\tilde{1}\}})_{2N}\\\cdots\cdots\\(\sqrt[N-1]{\{\tilde{1}\}})_{NN}\end{pmatrix}\right\}} \tag{2.6}$$

where, for the sake of simplicity, the term $\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}$ stands for $\{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$.
The same Ordinal Matrix, in addition, may always be represented in a more synthetic form by means of one sole symbol, by adopting the following synthetic notation

$$\{(\sqrt[N-1]{\{\tilde{1}\}})_{ij}\}^{\uparrow\{\tilde{N}/\tilde{N}\}} \tag{2.7},$$

where the arrow "↑" explicitly reminds us that the Ordinality $\{\tilde{N}/\tilde{N}\}$ has always to be considered as being a particular form of *Over-Ordinality*.

In this way the explicit solution to the first Fundamental Eq. (1) can synthetically be expressed as follows

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{\alpha}_{12}(t) \oplus \tilde{\lambda}_{12}(t)\} \circ (\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}} \tag{2.8}$$

Consequently, if such a solution is introduced into the Global Feed-Back Process represented by the second Fundamental Equation (2), the latter transforms into a typical Riccati's Equation of *Ordinal Nature*, whose explicit solution is given by

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{B}(t)\} \circ (\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}} \tag{2.9},$$

where

$$\tilde{B}(t) = \left\{ \begin{pmatrix} \oplus \tilde{A}(t) \\ \Theta \tilde{A}(t) \end{pmatrix}, \begin{pmatrix} \Theta \tilde{A}(t) \\ \oplus \tilde{A}(t) \end{pmatrix} \right\} \tag{2.10}$$

and

$$\tilde{A}(t) = \{\{\tilde{\alpha}_{12}(0)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(0)\}^{\{\tilde{2}/\tilde{2}\}}\} \circ (\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}\}^{(\tilde{2}/\tilde{2})} \oplus \ln(\tilde{c}_1 \oplus \{\tilde{c}_2, t\}) \tag{2.11},$$

in which the term $\ln(\tilde{c}_1 \oplus \{\tilde{c}_2, t\})$ accounts for the *origin and habitat conditions* of the Feed-Back Equation and, at the same time, also represents an *Over-Ordinality* contribution specifically due to the same Feed-Back Process.
Equation (A9) then represents the Explicit "Emerging Solution" to the Maximum Ordinality Principle, formulated in two "Incipient" Differential Equations ((1) and (2)), considered as being a Whole.

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