La Derivata “*Incipiente*” e L’ “*Irriducibile Eccedenza*” della Qualità

Introduzione

In questa *Prima Sezione*, specificamente dedicata alla Derivata “*Incipiente*”, abbiamo ritenuto opportuno evidenziare, rispettivamente:
a) le *Ragioni Gnoseologiche* che sono alla base della introduzione della Derivata “Incipiente”
b) il *Processo Storico-Scientifico* che ha “guidato” alla sua introduzione
c) La sua “*Origine*” da un punto di vista più propriamente *Formale*.

A tal fine, riportiamo dapprima due paragrafi della memoria di Gainesville 2016, in cui vengono chiaramente illustrati gli aspetti relativi ai punti a) e b), per poi riservare ad un nuovo paragrafo l’esposizione degli aspetti di cui al punto c).

1. “Emerging Quality” of Self-Organizing Systems and Consequential Adoption of New Mental Categories

The expression “*Emerging Quality of Self-Organizing Systems*” refers to the fact that Self-Organizing Systems always show an unexpected “*excess*” with respect to their phenomenological premises. So that they usually say: “*The Whole is much more than its parts*”.
Such an “excess” can be termed as *Quality* (with a capital Q) because it cannot be understood as being a simple “property” of a given phenomenon. This is because it is *never reducible* to its phenomenological premises in terms of traditional mental categories: *efficient causality*, *logical necessity*, *functional relationships*.
This evidently suggests a *radically new* gnosiological perspective, which corresponds to recognize that: “*There are processes, in Nature, which cannot be considered as being pure* “*mechanisms*”.
This also leads, *in adherence*, to the adoption of “*new mental categories*”[1] and, correspondently, to the development of a completely *new formal language*, so that the description of Self-Organizing Systems might result as being faithfully conform to their “Emerging Quality”.

2. The Progressive Development of an Appropriate Formal Language

L. Boltzmann was the first who attempted at describing Self-Organizing Systems in more appropriate formal terms, by proposing the adoption of a new Thermodynamic Principle: The Principle of Maximum Exergy *Inflow* to the System (Boltzmann 1886).
Some years later, A. Lotka (1922-1945) reformulated such a Principle in the form of: The Principle of Maximum Exergy *Flow through* the System (Lotka, 1922a,b, 1945).
Both such attempts were not perfectly successful, because still based on the concept of Exergy, which is a quantity that is strictly pertaining to Classical Thermodynamics. Consequently, it re-proposes the concepts of *efficient causality, logical necessity, functional relationships*.
A really *new formal language* only appears with H. T. Odum, with the genial introduction of Emergy (*Em*), defined as Exergy (*Ex*) by Transformity (*Tr*)

$$Em = Ex \cdot Tr \qquad (1).$$

Equation (1) clearly shows that Emergy is *still* based on “Exergy”. However:
i) *Quality Factor Tr* “*Transforms*” *Ex* into a *new physical quantity*: Emergy;
ii) The latter in fact is not defined in “functional terms”, but only by “*assignation Rules*” (Brown and Herendeen, 1996);
iii) This is precisely because *Tr* is expressed by means of a *non-conservative Algebra*;
iv) Thus the output “excess” of the three Fundamental Process (Co-Production, Inter-Action, Feed-Back) is always understood as being “irreducible” to its specific inputs in *mere functional terms*.
This means that Emergy is able to represent the “Emerging Quality” of Self-Organizing *Processes.* Consequently, the general enunciation of the *Maximum Em-Power Principle* (Odum 1994a,b,c) can *equally be referred*, at a phenomenological level, to the *corresponding maximization tendency* of the “Emerging Quality” on behalf of *Self-Organizing Systems*.

The Maximum Em-Power Principle, however, had not a corresponding formulation under *variable conditions*. On the other hand, such a formulation could not be given in terms of the Traditional Differential

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[1] These “*new mental categories*” can no longer be termed as “pre-suppositions”, because they are not defined “a priori” (as in the case of Traditional Approach). In fact, they are chosen only “a posteriori”, on the basis of the “Emerging Quality” previously recognized. “*Generative Causality*”, in fact, refers to the *capacity* of a Self-Organizing System to manifest an “irreducible excess”; “*Adherent Logic*”, correspondently, refers to the capacity of our mind to draw “*emerging conclusions*”. That is, “conclusions” whose information content is much higher than the information content corresponding to their logical premises, although persistently “adherent” to the latter. “Ordinal Relationships”, in turn, refer to particular relationships of *genetic nature*, which will be illustrated in more details later on, in the case of a Co-Production Process.

Calculus, because traditional derivatives, as a consequence of their conceptual basic presuppositions (see Tab. 1), are not properly apt to represent the "*generative*" behavior of "Self-Organizing Systems".

This is why, in order to achieve an appropriate mathematical formulation of the Maximum Em-Power Principle, I introduced the concept of "*Incipient Derivative*", defined as

$$\left(\frac{\tilde{d}}{\tilde{d}\,t}\right)^{\tilde{q}} f(t) = \underset{\tilde{\Delta} t:0\to 0^+}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}\, t}\right)^{\tilde{q}} \circ f(t) \qquad \text{for} \quad \tilde{q} = \tilde{m}/\tilde{n} \qquad (2).$$

A definition which clearly shows that the "*Incipient Derivative*" is not an "operator", like the traditional derivative (d/dt), but it could be termed as a "*generator*", because it describes a Process "*in its same act of being born*" (Giannantoni 2001a,b, 2002, 2004a,b, 2006, 2008a, 2010a).

The Mathematical Formulation of the M. Em-P. Principle in terms of *Incipient Derivatives* was preliminarily given in (Giannantoni 2001b) and, in a more articulated form, in a specific book co-financed by the Center for Environmental Policy (Giannantoni 2002).

During the successive eight years (2002-2010), such a mathematical formulation was applied to several Disciplines, such as *Classical Mechanics*, *Quantum Mechanics*, *General Relativity*, *Chemistry*, *Biology*, *Economics and the corresponding results were reunited in two books* (titled: *"Lightness of Quality"* (Giannantoni 2007) *and "Ascendency of Quality*" (Giannantoni 2008b).

At the end of this wide range of applications, I realized that it was possible to give a more general formulation of the Maximum Em-Power Principle, in the form of the "*Maximum Ordinality Principle*" (Giannantoni 2010a).

3. La Derivata "Incipiente", e gli associati Concetti di *Generatività* e *Ordinalità*

Al fine di illustrare come la Derivata "Incipiente" sia intimamente relata ai concetti di *Generatività* e *Ordinalità* dei Sistemi Auto-Organizzanti, riportiamo integralmente, qui di seguito, il paragrafo 2.4 del capitolo "Soluzioni Emergenti", tratto dal File "*La Qualità e il Principio di Massima Ordinalità*".

A seguire, riportiamo anche i primi lavori in cui "appare" la Derivata "Incipiente", per mostrare anche come, sin dall'inizio, tale concetto di Derivata fosse intimamente associato i concetti di *Generatività* e *Ordinalità*.

2.4 La *definizione formale* di derivata "incipiente"

Questo tipo di derivata è stata denominata "*incipiente*" perché mira a descrivere i Processi nel loro "sorgere", come esito di una loro specifica *attività generativa*. In altri termini, focalizza l'attenzione sui vari "output" (del Processo considerato) nell'atto stesso in cui questi sono "generati" dal Processo stesso, per evidenziarne la corrispondente "Eccedenza".

"La *prima apparizione* della Derivata Incipiente"

La definizione formale di Derivata Incipiente, già presentata in (Giannantoni 2001a,b, 2002, 2004a,b, 2006, 2008a) e richiamata anche, per alcuni suoi particolari caratteri, in Appendice 1 e in Appendice 4 dello stesso File precedentemente ricordato, viene qui ripresentata nei sui caratteri essenziali.

La *derivata incipiente* non è altro che la fedele rappresentazione, in termini formali, dei tre Presupposti fondamentali precedentemente ricordati: *Causalità Generativa*, *Logica Aderente*, *Relazioni Ordinali*.

Pertanto, diversamente dalla derivata *tradizionale*, che "proietta" su un qualsiasi Processo una visione *a-priori*, l'introduzione della derivata "incipiente" apre invece all'adozione di un linguaggio formale che cerca di "accogliere", per poi "descrivere", le novità "emergenti" da un qualsiasi Processo.

E' sulla base di questa definizione, infatti, che è stato possibile (come del resto già anticipato) dapprima reinterpretare in senso "dinamico-generativo" i tre Processi Fondamentali di Odum, e fornire quindi la formulazione matematica del Maximum Em-Power Principle.

Ma nel contempo essa è anche il fondamento della successiva generalizzazione formale di questo Principio, nella sua riformulazione come *Principio di Massima Ordinalità*.

La definizione formale di "Derivata Incipiente" è la seguente

$$\left(\frac{\tilde{d}}{\tilde{d}\,t}\right)^{\tilde{q}} f(t) = \underset{\tilde{\Delta} t:0\to 0^+}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}\, t}\right)^{\tilde{q}} \circ f(t) \qquad \text{for} \quad \tilde{q} = \tilde{m}/\tilde{n} \qquad (1.1),$$

da cui si riconosce facilmente che:

i) non è un "operatore", come la tradizionale derivata (d/dt), ma può più propriamente denominarsi un "*generatore*", perché descrive un Processo nel suo stesso atto "generativo";

ii) la sequenza dei simboli viene ora interpretata *da sinistra a destra* e, nel contempo, ciascun simbolo ha un *significato profondamente diverso* da quello tradizionale;

iii) questi, infatti, non rappresentano più "tre operazioni" distinte, ma un *unico e solo processo generativo*;

iv) il simbolo $\tilde{Lim}$, infatti, non indica più un "limite" matematico, ma sta a rappresentare una sorta di "finestra" o di "soglia" (dal Latino "*Limen*"); cioè proprio quella "finestra" (o, meglio, quella "prospettiva") secondo cui osserviamo e descriviamo il fenomeno in esame come un *Processo Generativo* inteso come "*Unum*";
v) il concetto $\Delta t \to 0$ viene ora non solo "invertito" nel suo senso "evolutivo", ma viene ad assumere un significato completamente diverso. Infatti, trascritto ora nella forma esplicita $\tilde{\Delta} t : 0 \to 0^+$, indica non solo l'istante iniziale della nostra registrazione fenomenologica, ma anche, più propriamente, l' "origine" (in senso etimologico) di qualcosa di nuovo *che* (*appunto*) *sta nascendo*;
vi) l' "operatore" Δ, che nella definizione di derivata tradizionale esprimeva la *variazione quantitativa* di una "funzione" $\Delta f(t)$, diviene ora un "generatore", rappresentato con $\tilde{\delta}$, per indicare così che la prospettiva adottata considera la variazione della *proprietà* $\tilde{f}(t)$, non solo in termini di quantità, ma anche, ed in particolar modo, in termini di Qualità (e ciò viene opportunamente ricordato, e intenzionalmente sottolineato, anche dal simbolo "tilde", specificamente adottato a tale scopo); .

vii) di conseguenza, il rapporto $\left(\frac{\tilde{\delta}-1}{\tilde{\Delta} t}\right)^{\tilde{q}}$ non indicherà più soltanto una variazione quantitativa nel tempo, ma indicherà, contemporaneamente, sia una variazione di *quantità* che una associata variazione di *Qualità*;
viii) in tal modo la derivata "incipiente" viene ad esprimere la *Generatività* del Processo attraverso la registrazione di quell' "Eccedenza" che si manifesta in "uscita" (per unità di tempo). Tale "uscita" sarà sicuramente caratterizzata da una sua *cardinalità*, ma anche, e soprattutto, da una specifica *Qualità*. Quest'ultima ci apparirà gerarchicamente ordinata, con modalità variabili in relazione al Processo considerato, ma sempre secondo una sua particolare *Ordinalità*, indicata con $\tilde{q}$ (dove q è un *qualunque numero razionale*), e che rappresenta appunto la *specifica tipologia* delle Relazioni (Ordinali) fra le varie *entità generate* dal Processo in esame. Proprio per questo la derivata "incipiente" potrà anche essere rappresentata più sinteticamente nella forma seguente

$$\left(\frac{\tilde{d}}{\tilde{d} t}\right)^{\tilde{q}} \tilde{f}(t) \qquad (1.2).$$

la quale, proprio sulla base di quanto precedentemente esposto, nel seguito di questo lavoro, e con riferimento ad un qualsiasi Processo, potrà essere interpretata secondo il suo significato più proprio di *Generatività di Ordinalità* $\tilde{q}$.
Ed è proprio questo concetto che consentirà di passare ad un'analisi più approfondita del Principio di Massima Ordinalità (trattato nella Terza Sezione) e di riconoscere così la *particolare novità* rappresentata delle "Soluzioni Emergenti" che da esso si originano.

4. L' "*Origine*" della Derivata "Incipiente" da un punto di vista *Formale*

Da un punto di vista strettamente "formale", la Derivata "Incipiente" *trae origine* dalla *inversione genere-specie* nella "tradizionale" definizione di derivata.
Infatti, come chiaramente espresso sin dalla sua "prima apparizione" (nel "The Maximum Em-Power Principle as the Basis for Thermodynamics of Quality", Cap. 14, Appendice 8, più oltre integralmente riportata):

Now we may ask: what happens if we interpret the sequence of symbols in expression (14.8.1) according to the same order as they are written (*that is from left to right*)*?*

Tale "inversione", infatti, dà origine ad un nuovo Concetto "linguistico-formale", che ha poi consentito (come peraltro già anticipato) la Formulazione Matematica, in termini del tutto generali, del Maximum Em-Power Principle, proposto da H.T. Odum nel 1994, che si è così rivelato come un *Principio Termodinamico* del tutto *innovativo* (come verrà chiaramente e ampiamente illustrato nella Seconda Sezione di questo Lavoro).

In questa Sezione, invece, specificamente dedicata alla "*Eccedenza* della Qualità", oltre alla definizione di Derivata "Incipiente" così come appare, *per la prima volta*, in Appendice al Volume "The Maximum Em-Power Principle as the Basis for Thermodynamics of Quality" (SGE Editoriali Padova, Novembre 2002), riportiamo anche, per ragioni di completezza circa l' "origine formale" di questo concetto, i seguenti due articoli:

a) "*The Problem of the Initial Conditions and Their Physical Meaning in Linear Differential Equations of Fractional Order*" (del 2001), in cui vengono presentati i presupposti formali della Derivata "Incipiente". Articolo preliminarmente presentato al *Third Workshop on "Advanced Special Functions and Related Topics in Differential Equations" - June 24-29, 2001 - Melfi* (*Italy*), e poi pubblicato su Applied Mathematics and Computation 2003; (141): 87-102;
b) Ed un successivo articolo (del 2004), "*Mathematics for Generative Processes: living and non-living Systems*", (Lovanio September 25-28, 2004), anche questo pubblicato su Applied Mathematics and Computation 2006; (189): 324-340.
A tal riguardo, per rispetto delle norme sull'Editoria, di quest'ultimi due lavori pubblichiamo solo i relativi Abstracts.

Iniziamo allora con il riportare (come anticipato):

4.1 "La *Prima Apparizione* della Derivata Incipiente"

Rif. Appendice al Volume:
"The Maximum Em-Power Principle as the Basis for Thermodynamics of Quality"

Chapter 14. ***Appendix 8. Generative Systems Dynamics and adherent "incipient" derivatives***
Such a different perspective starts from the consideration of the fact that the traditional definition of the derivative of a function $f(t)$ *given in Mathematical Analysis*

$$\lim_{\Delta t \to 0} \frac{\Delta}{\Delta t} f(t) \qquad (14.8.1)$$

*may be considered as being an "*a posteriori*" definition* (*e.g., let us think of the definition of velocity*)*. In fact, although it is usually* read *from left to right, it is vice versa* interpreted *from right to left. In other words its meaning is based on a* reverse priority *of the order of the three elements that constitute its definition: i*) *the concept of function* (*which is assumed to be a primary concept*)*; ii*) *the incremental ratio* (*of the supposedly known function*)*; iii*) *the operation of limit* (*referred to the result of the previous two steps*)*.*
Now we may ask: what happens if we interpret the sequence of symbols in expression (14.8.1) according to the same order as they are written (*that is from left to right*)*?*

Such a direct perspective gives rise to a new concept of derivative (*indicated by* $\frac{\tilde{d}}{\tilde{d}\, t}$ *and defined in Appendix 9*) *which can be named "*incipient*"* (*or "*spring*"*) *because of some special characteristics that will be illustrated through the derivative of the exponential function* $e^{\varphi(t)}$*, which now gives*

$$\frac{\tilde{d}^n}{\tilde{d}\, t^n} e^{\varphi(t)} = \left(\frac{\tilde{d}}{\tilde{d}\, t} \varphi(t) \right)^n \cdot e^{\varphi(t)} = \left(\tilde{\varphi}' \right)^n \cdot e^{\varphi(t)} \qquad (14.8.2).$$

Such a result is always formally different from the one obtainable through traditional ordinary derivatives, even when both results coincide numerically (*that is, for any order derivative, if* $\varphi(t) = \alpha t + \beta$*; otherwise, if* $\varphi(t)$ *is a non-linear function, only in the case of first-order derivative*)*. Consequently the adopted symbology reminds us of the main differences: i*) *the resulting expression refers to a* virtual *evolution, which may also become a real evolution, but only in correspondence with particular boundary conditions; ii*) *the comprehensive structure of Eq. (18.6.2) reminds us that the obtained result is due to a "*generating process*", the virtual* (*evolutive*) *possibilities of which are delineated in terms of its intrinsic* genetic characteristics $(\tilde{\varphi}')^n$ *, which are essentially due to both the* generator $\frac{\tilde{d}^n}{\tilde{d}\, t^n}$ (*understood as a* prior *"operator"*) *and the "fertile" co-operation of the considered "*generating*" function* $e^{\alpha t}$ *; iii*) *thus the final result represents an evolutive modality which is completely new with respect to the original function: it is not seen now as a "necessary" consequence* (*as in the case of operators interpreted* a posteriori) *but, because of the a priori interpretation of operators, it is conceived as an "adherent" consequence of its "generation" modalities: all the various functions resulting from the "generating process" represented by Eq. (14.6.2), for* $n \in N$ *, are similar to* harmonic evolutions *which are in "*resonance*"* (*as in a "musical chord"*) with the original function *and at the same time* with each other.

***Appendix 9. Definition of "incipient" or "spring" derivative* (of integer and fractional order)**

The definition of the "incipient" derivative, first referred to any integer n*, is given by* (*see also Giannantoni 2001d)*

$$\frac{\tilde{d}^n}{\tilde{d}\,t^n} f(t) = \tilde{\lim}_{\tilde{\Delta} t:0\to 0^+} \cdot \left(\frac{\tilde{\delta}-1}{\tilde{\Delta} t} \right)^{\tilde{n}} \cdot f(t) \qquad (14.9.1)$$

where the symbol $\tilde{\delta}$ *represents an "operator" that generates a translation of a function, that is*

$$\tilde{\delta}\, f(t) = f\left(t + \tilde{\Delta}\, t \right) \qquad (14.9.2),$$

which has the following characteristics: i) *the time variation* $\tilde{\Delta}\, t$ *can also be real, but in general it is understood as being virtual* (*and the associated symbol* $\tilde{\Delta}$ *reminds us of such an assumption*)*; ii*) *the symbol* $\tilde{\delta}\, f(t)$ *is not only the representation of the second side of Eq. (14.9.2), because the "operator"* $\tilde{\delta}$ is prior *with respect to* $f(t)$*: it is the one that originates such a virtual translation; iii*) *the "operator"* $\tilde{\delta}$ *may be thus better named as "*generator*" because, according to definition (14.9.2), it "acts" as generator of a translation; iv*) *the name "generator" also reminds us that it acts in combination with something else:* $\tilde{\delta}$ *is in fact the* prior *"principle",* $f(t)$ *is the* posterior *"principle", and* $f\left(t + \tilde{\Delta}\, t \right)$ *is what "rises" from the combination of both. Such a result* (*or "product"*) is something new, *but at the same time it retains the main genetic characteristics of its generating "principles".*

Analogous considerations can be made with respect to the "operator" $\left(\frac{\tilde{\delta}-1}{\tilde{\Delta} t} \right)^{\tilde{n}}$.

Finally, the operation of limit ($\tilde{\lim}_{\tilde{\Delta} t:0\to 0^+}$ *) is here also considered as a* prior operator *with respect to those that follow it in Eq. (14.9.1) but, at the same time, it is posterior to the very primary operation: the passage from the time* t*, initially prefixed, to the virtual time*

$$\tilde{\delta}\, t = t + \tilde{\Delta} t \qquad (14.9.3)$$

as a consequence of a virtual translation generated by the "generator" $\tilde{\delta}$*. Such an operation is represented by the symbol* $\tilde{\Delta}\, t : 0 \to 0^+$ *to remind us that our concept of "limit" is a "*spring-concept*": it is the "source" of what rises as a consequence of an infinitesimal virtual variation, immediately after a given time* t*, which in turn activates the sequence of the successive "generators" in its "spring-perspective".*

Definition (14.9.1) also implies

$$\frac{\tilde{d}^n}{\tilde{d}\,t^n} e^{\varphi(t)} = \left(\frac{\tilde{d}}{\tilde{d}\,t} \varphi(t) \right)^{\tilde{n}} \cdot e^{\varphi(t)} = \left(\tilde{\varphi}' \right)^n \cdot e^{\varphi(t)} \qquad (14.9.4)$$

so that, for any function $f(t) = e^{\ln f(t)}$*, we consequently have*

$$\frac{\tilde{d}^n}{\tilde{d}\,t^n} f(t) = \frac{\tilde{d}^n}{\tilde{d}\,t^n} e^{\ln f(t)} = \left(\frac{\tilde{f}'(t)}{f(t)} \right)^{\tilde{n}} \cdot e^{\ln f(t)} = \left(\frac{\tilde{f}'(t)}{f(t)} \right)^{\tilde{n}} \cdot f(t) = \left(\tilde{\beta}_f(t) \right)^n \cdot f(t) \qquad (14.9.5),$$

where the factor of similarity $\tilde{\beta}_f(t)$ (*generally depending on time*) *acts as a new "generator" in Eq. (14.9.5), similarly to* $\tilde{\varphi}'(t)$ *in Eq. (14.9.4). In general it can be either a scalar quantity or a vector or even a matrix. This evidently depends on the function under consideration.*
Eq. (14.9.5) can easily be extended to any $q \in Q$ *and applied to the most common functions in Mathematical Analysis* (*such as, for instance, analytical functions*).

Third Workshop on "Advanced Special Functions and Related Topics in Differential Equations" - June 24-29, 2001 - Melfi (*Italy*). Applied Mathematics and Computation 2003; (141): 87-102

4.2 The Problem of the Initial Conditions and Their Physical Meaning in Linear Differential Equations of Fractional Order

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Abstract. The well-known approaches to fractional differential equations (such as those proposed by Grunwald-Post and Riemmann-Liouville) imply, as initial conditions, the evaluation of fractional derivatives of the unknown function. From an applicative point of view, an extensive use of the corresponding and well-defined theoretical solutions would require an appropriate interpretation of the physical meaning of such initial conditions. On the other hand some authors propose to overcome the problem by introducing a different definition of fractional derivative which only requires the evaluation of integer-order derivatives of the un-known function at the lower extreme of the definition interval.
The problem we are concerned with is faced here in the light of an «intensive» concept of fractional derivative which has been elaborated in order to give differential bases to Emergy Algebra. Such an approach, apart from some peculiar novelties, is potentially able to solve the above-mentioned problem by suggesting a clear and meaningful physical interpretation of the initial conditions.
As far as the main novelties are concerned it is worth mentioning the following: i) fractional differentiation generates a *multiplicity* of derivatives (instead of a *unique* derivative) whose number is strictly dependent on the order of derivation; ii) the exponential function results as an «invariant» (in module) with respect to the order of differentiation; iii) the exponential function is therefore able to play a «hinge» role in solving *fractional* differential equations similar to the role it plays in the case of ordinary differential equations.
As a consequence of the above-mentioned properties it is possible to assert that: a time differential problem described by *one fractional* differential equation generates new "special" functions (the "binary", "ternary", "quaternary" functions and so on) which can be interpreted as being the mathematical description of the evolution of a *unique* System, made up of a prefixed number of *parts*, which are in turn so strictly related to each other that they form *one sole entity*. Consequently, the pertaining fractional initial conditions directly refer to and describe the physical *inter-actions* between *the parts* of the System at the given initial time.

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4.3 Mathematics for Generative Processes: living and non-living Systems

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Abstract. The traditional Differential Calculus often shows its limits when describing living Systems. These in fact present such a richness of characteristics that are, in the majority of cases, much wider than the description capabilities of the usual differential equations. Such an aspect became particularly evident during the research (completed in 2001) for an appropriate formulation of Odum's *Maximum Em-Power Principle* (proposed by the Author as a possible *Fourth* Thermodynamic Principle). In fact, in such a context, the particular *non-conservative* Algebra, adopted to account for both Quality and quantity of generative processes, suggested we introduce a faithfully corresponding concept of "derivative" (of both integer and fractional order) to describe dynamic conditions however variable. The new concept not only succeeded in pointing out the corresponding differential bases of all the rules of Emergy Algebra, but also represented the preferential guide in order to recognize the most profound *physical nature* of the basic processes which mostly characterize self-organizing Systems (co-production, co-injection, inter-action, feed-back, splits, etc.).

From a mathematical point of view, the most important novelties introduced by such a new approach are: i) the derivative of any integer or fractional order can be obtained *in-dependently* from the evaluation of its lower order derivatives; ii) the exponential function plays an extremely *hinge role*, much more marked than in the case of traditional differential equations; iii) wide classes of differential equations, traditionally considered as being non-linear, become "intrinsically" linear when reconsidered in terms of "incipient" derivatives; iv) their corresponding *explicit* solutions can be given in terms of new classes of functions (such as "binary" and "duet" functions); v) every solution shows a sort of "persistence of form" when representing the product *generated* with respect to the agents of the *generating* process; iv) and, at the same time, an intrinsic "genetic" *ordinality* which reflects the fact that any product "generated" is *something more* than the sum of the generating elements. Consequently all these properties enable us to follow the evolution of the "product" of any *generative* process from the very beginning, in its "rising", in its "incipient" act of being born. This is why the new "operator" introduced, specifically apt when describing the above-mentioned aspects, was termed as "incipient" (or "spring") derivative.

In addition, even if the considered approach was suggested by the analysis of *self-organizing living* Systems, some specific examples of non-living Systems will also be mentioned. In fact, what is much more surprising is that such an approach is even more valid (than the traditional one) to describe non-living Systems too. In fact the resulting "drift" between traditional solutions and "incipient" solutions led us to reconsider the phenomenon of Mercury's precessions. The satisfactory agreement with the astronomical data suggested, as a consequential hypothesis, a different interpretation of its *physical origin*, substantially based on the *Maximum Em-Power Principle*.

Keywords: Self-organizing Systems, Linear and Non-linear Differential Equations, Integer and Fractional "Incipient" Derivatives, Explicit Solutions